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A NOTE ON THE ANCHORING EFFECT OF EXPLICIT INFLATION TARGETS

JAN LIBICH
La Trobe University
and
CAMA

Empirical literature provided convincing evidence that explicit (i.e., legislated) inflation targets anchor expectations. I propose a novel game theoretic framework with generalized timing that allows us to formally capture this beneficial *anchoring effect*. Using the framework I identify several factors that influence whether and how strongly expectations are anchored, namely (i) the public's cost of decision making, (ii) the public's inflation aversion, (iii) the slope of the Phillips curve, (iv) the magnitude of supply shocks, (v) the degree of central bank conservatism, and under many (but not all) circumstances, (vi) the explicitness of the inflation target.

Keywords: Anchoring Effect, Rational Inattention, Endogenous Timing, Explicit Inflation Targets

1. INTRODUCTION

Private expectations of the future play a central role in the optimal setting of monetary policy, as well as its outcomes. The extent to which the policy's design can affect expectation formation has been a matter of debate. Several recent empirical papers contributed to this debate by showing that in countries with an explicit (i.e., legislated) inflation target expectations are better anchored.[1]

Expectations that are anchored—"*relatively insensitive to incoming data*" [Bernanke (2007)]—are of interest because they give central bankers more leverage over the real interest rate, and hence make their stabilization efforts more effective. For this reason Kohn (2008) argued: "*. . . anchoring is critical*"—see also Mishkin (2007).

This paper offers a formal model of how explicit inflation targets anchor expectations, i.e., why they may make private agents inattentive. It postulates a novel

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game theoretic framework that generalizes the timing of the players' actions. Specifically, the policy maker and the public will no longer necessarily move every period and/or in a simultaneous fashion, which has been the case in most settings. Instead, the players will be moving with a certain *constant frequency* that can (i) differ across the players and (ii) be *endogenous*.

The public in the model is "economically rational," and hence it optimally chooses the frequency with which it updates expectations—based on a cost/benefit calculation. This frequency affects the public's utility in three respects, which I refer to as (i) the *decision-cost motive*, (ii) the *accuracy motive*, and (iii) the *monitoring motive.*

In terms of (i), I assume that the public faces some cost of decision making, e.g., gathering and processing information. Naturally, this cost is increasing in the frequency of updating expectations. As such, it constitutes a reason for the public to be *rationally inattentive* [optimally choose to update expectations *less frequently*—see, e.g., Reis (2006)], and hence for expectations to be anchored.

In contrast, the two motives (ii) and (iii) go in the opposite direction and provide the public with reasons for updating expectations *more frequently*. The (short run) accuracy motive refers to the public's effort to correctly respond to shocks in real time and minimize expectation errors. I show that this motive can identify a number of variables that determine the degree to which expectations are anchored, but it cannot explain the anchoring effect of explicit inflation targets.

The (long run) monitoring motive can do so. It refers to the public's attempt to discourage the policy maker from deviating from the optimal long run inflation level. By frequent updating, the public can "punish" the policy maker and reduce his output gain from small surprises. Because an explicit inflation target can be "reconsidered" less frequently than an implicit one, the public's punishment lasts longer and can thus be less frequent. Therefore, the optimal frequency of expectation updating is a decreasing function of the explicitness of the target—i.e., the *anchoring effect* occurs.

The analysis is complemented by Libich (2009), who formally shows how anchored expectations translate into an improvement in macroeconomic stabilization.[2]

2. MACROECONOMIC MODEL

2.1. Economy

For illustration, I use a familiar New Keynesian model—a simplified version of Clarida, Gali, and Gertler (1999). Two equations—a Phillips curve and an IS curve—describe the economy, but we will need only the former for our purposes:

$$\pi_t = \lambda x_t + e_t + u_t, \tag{1}$$

where $\lambda > 0$, π denotes inflation, e denotes inflation expectations, x expresses the output gap, and t denotes time. For algebraic convenience I assume the periods to be discrete steps of an arbitrarily small length. The timing of e will be endogenous and is described below, but in any case the public is, like the policy maker, *forward looking* and acts rationally. Both players also have common knowledge of rationality and complete information about the economy and the structure of the game. The variable u is an inflation shock with a zero mean and variance σ_u^2. The shock is observable in real time by the players that can move in that period.[3]

2.2. Players

The preferences of the players are as follows (I will assume out discounting for simplicity). The policy maker has the standard quadratic one-period utility function, namely

$$U_t^g = -\alpha(x_t - x^T)^2 - (\pi_t - \pi^O)^2, \tag{2}$$

where π^O is the socially optimal low inflation level. The positive parameter α expresses the degree of the central bank's conservatism, and I will restrict my attention to the realistic cases $\alpha \in (0, 1)$.[4] The output gap target is $x^T \in \mathbf{R}$, i.e., the output target itself may be above, below, or equal to potential output. I will first consider the case $x^T = 0$ and then examine $x^T \neq 0$. The public's one-period utility function is the following:

$$U_t^p = -(\pi_t - e_t)^2 - C_\pi - C_e, \tag{3}$$

where the three components will underlie the three motives discussed above. The first element is the *inaccuracy cost*—a common representation of rational expectations. The C_π element is an *inflation cost*, and the C_e variable is a *decision making cost*.

2.3. Solution

In a conventional one-shot (one-period) game our model yields outcomes analogous to those of Clarida et al. (1999). To demonstrate this, set up the Lagrangian and derive the familiar targeting rule under discretion:

$$\pi_t - \pi^O = -\frac{\alpha}{\lambda}(x_t - x^T). \tag{4}$$

Substituting (4) into the Phillips curve and imposing rational expectations then implies

$$\pi_t^* = \pi^O + \frac{\alpha}{\lambda}x^T + \frac{\alpha}{\alpha + \lambda^2}u_t \quad \text{and} \quad x_t^* = -\frac{\lambda}{\alpha + \lambda^2}u_t, \tag{5}$$

which are the standard values of inflation and the output gap in equilibrium (all equilibrium values will be denoted by an asterisk throughout).

2.4. Two Policy Instruments

In modern macroeconomic models the central bank's instrument is the *short-term interest rate* i, which determines the level of inflation in each period, π. Likewise in my model, but I suppress the demand side for parsimony.

In addition, there is another policy "instrument." The policy maker chooses the level of its *long run inflation target*, π^T. Long run expresses the fact that it is the policy maker's preferred *average level* of inflation, $\bar{\pi}$ (all averages will be denoted by a bar). Therefore, it does not need to be achieved each and every period; it needs only to be delivered *on average* over the medium–long run (business cycle).

Specifically, in a certain period the central bank may, in order to stabilize output, optimally select an inflation rate that deviates from its inflation target, $\pi_t^* \neq \pi^T$. Therefore, when stating that the long run inflation *target is achieved* or *deviated from*, our meaning is always in an *average* (long run) sense.[5] Such mutual consistency of the short run and long run instruments, generally present in most macroeconomic models, can be seen in (5), where the supply shock does not affect the average (long run) values.

3. RATIONAL INATTENTION AND THE ANCHORING EFFECT

3.1. Timing of Moves

To generalize the timing of the standard repeated game but still keep the framework as comparable as possible, I will assume that the players move with a certain constant frequency. In terms of the central bank, I assume that it can adjust i—and hence π—every period, whereas it can adjust its inflation target π^T only every r^b periods. In interpreting r^b I will assume that because a more explicit target is more visible by the public, it can be less frequently reconsidered and altered. One can think of institutional (legislative) constraints or reputational consequences following frequent changes of the inflation target.[6] Naturally, these can be stronger if the target is explicit than if it is implicit; hence we have the following:

DEFINITION 1. *The variable r^b expresses the degree of* explicitness of the inflation target.

In terms of the public, it will update its expectations every r^p periods, whereby r^p will be endogenously determined (optimally selected by the public).[7] Let us define some terminology that will be used throughout.

DEFINITION 2. *The level r^{p*} optimally selected by the public expresses the* degree of rational inattention. *The public is* rationally inattentive *if r^{p*} is strictly positive (does not approach zero). Inflation expectations are* anchored *if (i) the public is rationally inattentive and (ii) expectations are on average equal to the optimal inflation target, $\bar{e} = \pi^O$. In such case the variable r^{p*} also expresses the degree of* anchorness of expectations.

The definition implies that whereas anchored expectations in my model always imply some degree of rational inattention, the reverse is not true. In both cases we have a strictly positive (and potentially large) r^{p*}, but in the latter case expectations may or *may not* equal the optimal inflation level on average.

DEFINITION 3. *An explicit inflation target will be said to have an* anchoring effect *if (i) expectations are anchored and (ii) the equilibrium degree of expectation anchorness,* r^{p*}, *is a nondecreasing function of the target's explicitness,* r^b.

To study the anchoring effect of explicit inflation targets effectively, let us assume that

$$\frac{r^b}{r^p} = \left\lfloor \frac{r^b}{r^p} \right\rfloor > 1, \tag{6}$$

where $\lfloor . \rfloor$ denotes the integer value (the so-called floor function). This purely technical restriction will ensure that the game is closer to the standard repeated game setup, as it features both *synchronized* (i.e., simultaneous) and *asynchronized* moves. It implies that (i) every r^b the public updates expectations and the central bank reconsiders the inflation target simultaneously and (ii) expectations can also get adjusted in between these synchronized moves because $r^b > r^p$. Combining these points implies that (iii) the *dynamic stage game* is r^b periods long and that it gets regularly repeated. I will denote the *horizon* of the game, i.e., the total number of periods by T.

Let us summarize the timing of moves:

(1) At the beginning of the game, in $t = 0$, the public chooses r^p—observing r^b.
(2) Still in $t = 0$ and observing r^p, r^b, and the current shock, the policy maker and the public make a synchronized first move of all their instruments $\{\pi(i), \pi^T, e\}$.
(3) The policy maker then reconsiders the interest rate (and hence inflation) every period and the long run inflation target every r^b periods. The public updates expectations every r^p periods. All these moves are made observing all past and current shocks, as well as all past moves of the opponent.
(4) The payoffs are accrued every period until period T after which the dynamic stage game finishes.

3.2. Three Motives in Terms of r^{p*}

The public's incentive to update expectations *less frequently*, which provides reasons for rational inattention and anchored expectations, is due to the associated cost of doing so, C_e. For example, Mankiw and Reis (2002) discuss the existence of costs related to *"changing wage contracts and information-gathering, decision making, negotiation and communication."*

The related body of literature assumes, either implicitly or explicitly, that this cost is a per-period fee increasing in the number of updating/processing. The same will be assumed in our model, $\partial C_e/\partial r^p < 0, \forall r^p$. To obtain a clear-cut and illustrative analytical solution I will use the following functional form:

$$C_e = \frac{c_e}{r^p}, \quad \text{where } c_e > 0. \tag{7}$$

In contrast, the public's incentive to update expectations *more frequently* is due to the two remaining elements in its objective function. The first—short run—reason is the *accuracy motive*, which works for any $x^T \in \mathbf{R}$ of the policy maker. In a period in which the public does not update expectations, it does not optimally react to the current shock (if any). Expectations will therefore be set incorrectly and deviate from actual inflation, which is costly to the public. A lower r^p will decrease the proportion of periods r^p/T with such inaccuracy cost and hence increase the public's utility.

The long run reason for updating expectations more frequently, the *monitoring motive*, is determined by two factors. First, the public is averse to deviations of long run inflation from the optimal long run level, which we called the inflation cost C_π. Let us postulate it as the following fixed per-period cost:

$$C_\pi = \begin{cases} c_\pi > 0 & \text{if } \bar{\pi} \neq \pi^O, \\ 0 & \text{if } \bar{\pi} = \pi^O. \end{cases} \tag{8}$$

The second driver of the monitoring motive is the fact that the policy maker's output target may differ from potential, $x^T \neq 0$. He then has an incentive to carry out inflation/deflation surprises to achieve its output objective. As these are costly to the public (increase C_π), the public may find it optimal to keep the policy maker in check and eliminate this incentive. It has a way of doing so: because the policy maker can adjust its long run inflation only every $r^b > r^p$ periods, the public can punish the policy maker for such behavior.[8]

The fact that the size (length) of the punishment is increasing in r^b/r^p implies that under a more explicit target (higher r^b), less frequent expectation updating (higher r^p) is required to deliver a punishment of the same magnitude and eliminate the policy maker's temptation (i.e., minimize C_π). The fact that the public also wants to economize on its C_e cost then implies that r^{p*} is an increasing function of r^b.

4. THE SHORT RUN ACCURACY MOTIVE

This section will deal with the public's trade-off between infrequent updating (and minimizing the decision making cost C_e) and frequent updating [and minimizing the inaccuracy cost $(\pi_t - e_t)^2$].

PROPOSITION 1. *Assume* no monitoring motive, $x^T = 0$. *The public is rationally inattentive and its expectations are anchored, whereby the* equilibrium anchorness *is decreasing in (i) the variance of shocks σ_u^2 and increasing in (ii) the cost of decision making c_e, (iii) the output sensitivity of inflation λ, (iv) the time* *horizon* T, *and (v) the policy conservatism* $1/\alpha$. *An explicit inflation target has, however,* no anchoring effect.

Proof. Under $x^T = 0$ there is no temptation to surprise inflate/deflate, and hence (5) shows that *average* inflation and expectations are at the O level throughout, $\bar{e}^* = \bar{\pi}^* = \pi^O$. It then follows from (8) that $C_\pi = 0$. The inaccuracy cost differs in periods in which expectations get updated [whose proportion over all periods is $(T - r^p)/T$] and those in which this does not happen (whose proportion is r^p/T). The public's *expected* period utility (denoted by EU_t^p) is therefore a weighted average of utilities from these two types of periods:

$$\mathrm{EU}_t^p = -\left[\frac{T - r^p}{T}0 + \frac{r^p}{T}\left(\frac{\alpha}{\alpha + \lambda^2}\sigma_u\right)^2\right] - 0 - \frac{c_e}{r^p}. \tag{9}$$

This summarizes the implications of (1) and (3) that in the updating periods the inaccuracy cost is zero, because expectations are set accurately, $e_t^* = \pi_t^*$, and that in the non-updating periods the cost $(\pi_t^* - e_t^*)^2$ is of the expected size $(\frac{\alpha}{\alpha+\lambda^2}\sigma_u)^2$. The latter is because the policy maker can adjust its short-term interest rate instrument every period, choosing the optimal level π_t^* from (5), whereas expectations will be preset at the long run component of π_t^* from (5), π^O, because the shock cannot be predicted.

Differentiating (9) with respect to r^p, setting equal to zero, and rearranging yields[9]

$$\hat{r}^p = \frac{\alpha + \lambda^2}{\alpha}\sqrt{\frac{2Tc_e}{\sigma_u^2}}. \tag{10}$$

The fact that $\hat{r}^p$ in (10) is a function of the five variables in Proposition 1 with the desired signs, but not a function of r^b, completes the proof. ∎

Note that all five determinants of the degree of anchorness work in the expected direction (e.g., larger shocks reduce it). Explicit targets play no anchoring role because without the policy maker's temptation to surprise, the public has no incentive to monitor.

5. THE LONG RUN MONITORING MOTIVE

The public has an incentive to monitor the policy maker only under $x^T \neq 0$.[10] The literature has identified several possible reasons for $x^T \neq 0$, such as (i) mismeasurement of potential output, (ii) market imperfections, or (iii) political economy reasons.

The public monitors to minimize its inflation cost C_π, i.e., reduce deviations of average inflation from the optimal long run inflation level. The monitoring motive is therefore about the *average level*, at which expectations are anchored. It was shown in (5) and discussed in Section 2.4 that the average level of inflation and expectations is unaffected by zero-mean shocks. This implies that we can, without loss of generality, separate the monitoring motive from the accuracy motive and examine the former by abstracting from shocks and short run deviations. Put differently, the setting of the policy maker's short run instrument $\pi(i)$ becomes superfluous in this section, because we know that on average it will be set to be consistent with the selected long run inflation target, π^T. This allows us to focus only on the π^T and e actions, whereby the latter should also be interpreted as choosing some *average* level, $\bar{e}$.

To better communicate the intuition of the anchoring effect, I will streamline the exposition of the rest of my analysis by making several assumptions. First, I normalize $\lambda = 1$ and $\pi^O = 0$ throughout. Second, in presenting the normal form of the game I truncate the long run action sets of the players to two *average* levels (the short run levels will, however, remain unrestricted). Specifically, I follow the standard truncation [see, e.g., Cho and Matsui (2005)] and choose the two levels of interest: one is the socially optimal level, O, and the other is the equilibrium (but potentially time inconsistent) long run level from (5), which I denote by S as suboptimal:

$$\pi^T \in \left\{\pi^O, \pi^S = \pi^O + \frac{\alpha}{\lambda} x^T\right\} \ni \bar{e}. \tag{11}$$

Note that under the considered $x^T \neq 0$, the O and S levels are different for all α.

5.1. Equilibrium of the (Standard) Static Stage Game

Let us first examine the outcomes of the standard *static* stage game (lasting one period), which are unaffected by r^b and r^p. The payoffs can be obtained using the macroeconomic outcomes, (1)–(3), with the truncation (11). Denoting them by $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$, the payoff matrix is then as in Table 1.[11]

Note that the payoffs of the public satisfy, for all parameter values,

$$\mathbf{w} > \mathbf{x} \quad \text{and} \quad \mathbf{z} > \mathbf{y}. \tag{12}$$

Equation (3) then implies that the public's static best response is always to choose the action level equivalent to the central bank's, i.e., set expectations in line with inflation. Further note that for all considered α the policy maker's payoffs satisfy

$$\mathbf{c} > \mathbf{a} \quad \text{and} \quad \mathbf{d} > \mathbf{b}. \tag{13}$$

The relationships in (12)–(13) imply that the standard static game has a unique Nash equilibrium, $(\pi^S, \bar{e}^S)$, which is Pareto-inefficient—inferior to $(\pi^O, \bar{e}^O)$. This is because π^S is the policy maker's strictly dominant strategy—due to $x^T \neq 0$. It will become apparent below that allowing for the players' actions to be infrequent, i.e., considering the *dynamic* stage game, may alter these outcomes.

5.2. Equilibrium of the Dynamic Stage Game

As explained in Section 3.1, the full game consists of a dynamic stage game r^b periods long that gets repeated (T/r^b times).

TABLE 1. The payoff matrix of the static stage game

		Public	
		$\bar{e}^O$	$\bar{e}^S$
Central bank	π^O	$\mathbf{a} = -\alpha x^{T^2}$; $\mathbf{w} = -C_e$	$\mathbf{b} = -\alpha(\alpha+1)^2 x^{T^2}$; $\mathbf{x}= -\alpha^2 x^{T^2} - C_e$
	π^S	$\mathbf{c} = (-\alpha^3 + \alpha^2 - \alpha)\, x^{T^2}$; $\mathbf{y} = -\alpha^2 x^{T^2} - c_\pi - C_e$	$\mathbf{d} = -\alpha(\alpha+1)\, x^{T^2}$; $\mathbf{z} = -c_\pi - C_e$

DEFINITION 4. *Any subgame perfect Nash equilibrium (SPNE) that has, on its equilibrium path, all players playing in all long run moves (i) the socially optimal O levels will be called* Ramsey*; (ii) the inferior S levels will be called* anti-Ramsey.

Obviously, in addition to these two *types of SPNE* that are symmetric, there may exist a number of other non-Ramsey SPNE with both the O and S levels. In search for the anchoring effect, I will focus on deriving conditions under which the *dynamic stage game* has (i) a *unique* SPNE that is (ii) of the Ramsey type and hence *Pareto-efficient.* Under these conditions repeating the game will not affect the set of SPNE (and the effective minimax values). The uniqueness condition also means that we can focus on pure strategies without loss of generality.

5.3. Results

The following result relates to the public's monitoring motive.

PROPOSITION 2. *Updating inflation expectations* more frequently *reduces the incentives of the central bank to carry out inflation/deflation surprises, if any. If the public updates expectations* sufficiently frequently, $r^p < \tilde{r}^p$, *then the long run inflation target is not deviated from (on average) even under* $x^T \neq 0$.[12]

Proof. We can restrict our attention to the r^b period dynamic stage game for reasons explained in Section 5.2. Solving backwards, i.e., taking both r^p and r^b as given, let us analyze the players' π^T and $\bar{e}$ actions. We know that the public will find it optimal to play the same action in all its asynchronized moves—they are all made under the same circumstances. The public's rationality and (3) imply that the optimal action selected in all these moves will be the static best response to the policy maker's initial (now observable) move, i.e., $\bar{e}^*_{t\in(0,r^b)} = \pi_0^T$.

Moving backwards, we now need to determine the policy maker's optimal play in $t = 0$. For the optimal inflation target to be *time consistent* (and for a Ramsey SPNE to exist), it is required that π_0^O be the best response to $\bar{e}_0^O$. This is guaranteed by the following condition:

$$\mathbf{a}r^b \geq \mathbf{c}r^p + \mathbf{d}(r^b - r^p). \tag{14}$$

Both the left-hand side (LHS) and the right-hand side (RHS) are derived assuming that the public plays $\bar{e}_0^O$. The LHS expresses the fact that if the policy maker chooses π_0^O, he will achieve the payoff **a** in all r^b periods. In contrast, the RHS describes the scenario of the policy maker playing π_0^S and initially getting output closer to his preferred level x^T through an inflation/deflation surprise and the **c** payoff. This however lasts only for r^p periods, after which the public switches to $\bar{e}^S$ and punishes the policy maker (with a **d** payoff) for the rest of the stage game. Substituting in the respective values $\{\mathbf{a}, \mathbf{c}, \mathbf{d}\}$ from the payoff matrix in Table 1 and rearranging yields

$$r^p \leq \frac{r^b}{2-\alpha}. \tag{15}$$

Although (15) ensures the *existence* of a Ramsey SPNE, we are interested in deriving conditions under which there is a *unique* Ramsey SPNE. This is to make sure the S level expectations never occur on the equilibrium path. For this to be the case, π_0^O must be a strictly dominant strategy; thus in addition to (15) it is required that π_0^O is the unique best response also to $\bar{e}_0^S$. We can think of this as the willingness of the central bank to carry out a disinflation even if it knows that the disinflation will lack credibility and will therefore be costly. The following condition, derived in the same way as (14) but assuming that the public plays $\bar{e}_0^S$, ensures this:

$$\mathbf{b}r^p + \mathbf{a}(r^b - r^p) > \mathbf{d}r^b. \tag{16}$$

Rearranging (16) and using the payoff matrix yields

$$r^p < \tilde{r}^p = \frac{r^b}{2+\alpha}. \tag{17}$$

Comparing the two conditions implies that (17) is stronger than (15) for all considered α. It is therefore the necessary and sufficient condition for *existence* and *uniqueness* of Ramsey SPNE.[13] If this condition holds then we obtain, for all α, r^b, and x^T, the long run inflation (target) at the socially optimal O level in all periods. ■

Let us summarize the outcomes of the game as a function of r^p and r^b. The proof implies that

(1) under $\frac{r^p}{r^b} < \frac{1}{2+\alpha}$ we have a unique SPNE, and it is of the Ramsey type;
(2) under $\frac{r^p}{r^b} > \frac{1}{2-\alpha}$ there is a unique SPNE, and it is of the anti-Ramsey type;
(3) under $\frac{r^p}{r^b} \in [\frac{1}{2+\alpha}, \frac{1}{2-\alpha}]$ there exist *multiple* SPNE, one of which is Ramsey, one of which is anti-Ramsey, and the rest of which are other non-Ramsey types with both O and S levels on the equilibrium path, for one or both players.[14]

PROPOSITION 3. *If the public's inflation cost is sufficiently large, $c_\pi \geq \tilde{c}_\pi$, then the public always chooses to update expectations sufficiently frequently, $r^{p*} = \min\{\hat{r}^p, \tilde{r}^p\}$, to uniquely ensure a Ramsey SPNE. Then an explicit inflation target has an* anchoring effect.

Proof. In its r^{p*} decision, the public solves backwards and takes into account the equilibrium outcomes in the later periods of the dynamic stage game derived earlier in this section. The public therefore compares its utility from choosing some $r^p > \tilde{r}^p$ (and hence getting the $\mathbf{z}$ payoff with $C_\pi = c_\pi$ from the anti-Ramsey SPNE) and from choosing some $r^{p*} \leq \tilde{r}^p$ (and getting the $\mathbf{w}$ payoff with $C_\pi = 0$ from the Ramsey SPNE).

Let us first consider the case in which $\hat{r}^p$ from (10) falls into this interval, i.e., $\hat{r}^p \leq \tilde{r}^p$. Setting up the inequality, this is true iff

$$r^b \geq \tilde{r}^b = \frac{(2+\alpha)(\alpha+\lambda^2)}{\alpha}\sqrt{\frac{2Tc_e}{\sigma_u^2}}. \tag{18}$$

In such a case we know that $r^{p*} = \hat{r}^p$, because this is the maximum utility level based on the accuracy motive, and the constraint $r^{p*} \leq \tilde{r}^p$ coming from the monitoring motive and ensuring $C_\pi = 0$ is automatically satisfied.

In the opposite case, $\hat{r}^p > \tilde{r}^p$, the monitoring motive and minimization of C_π, however, requires a more frequent (and hence more costly) updating than the optimal frequency implied by the accuracy motive alone. Therefore, for the public to update sufficiently frequently in such a case, $r^{p*} \leq \tilde{r}^p$, an extra condition must be satisfied (as C_e is decreasing in r^p the public will rationally only consider the highest r^p value in this interval, i.e., $\tilde{r}^p$). Specifically, for $U_t^p(r^p = \tilde{r}^p) \geq U_t^p(r^p = \hat{r}^p)$ in the presence of shocks it is required that

$$-\frac{\tilde{r}^p}{T}\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2 - \frac{c_e}{\tilde{r}^p} \geq -\frac{\hat{r}^p}{T}\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2 - \frac{c_e}{\hat{r}^p} - c_\pi.$$

Intuitively, the c_π cost has to be sufficiently high *relative* to the c_e cost to justify more frequent updating. Substituting in the $\tilde{r}^p$ and $\hat{r}^p$ values yields

$$\begin{aligned} c_\pi &\geq \tilde{c}_\pi \\ &= \frac{\left[\alpha r^b - (2+\alpha)(\alpha+\lambda^2)\sqrt{\frac{2Tc_e}{\sigma_u^2}}\right]\left[\alpha r^b \sigma_u^2(\alpha+\lambda^2)\sqrt{\frac{2Tc_e}{\sigma_u^2}} - (\alpha+\lambda^2)^2(2+\alpha)Tc_e\right]}{(2+\alpha)(\alpha+\lambda^2)^3 T r^b \sqrt{\frac{2Tc_e}{\sigma_u^2}}}. \end{aligned} \tag{19}$$

If this condition is satisfied we can summarize the equilibrium degree of expectation anchorness as follows:

$$r^{p*} = \begin{cases} \hat{r}^p = \sqrt{\frac{2Tc_e}{\sigma_u^2}\left(\frac{\alpha+\lambda^2}{\alpha}\right)^2} & \text{if } r^b \geq \tilde{r}^b, \\ \tilde{r}^p = \dfrac{r^b}{2+\alpha} & \text{if } r^b \leq \tilde{r}^b. \end{cases} \tag{20}$$

Noting that r^{p*} is a nondecreasing function of r^b completes the proof. ■

It is interesting to note that the target's explicitness increases expectation anchorness only up to a point, $\tilde{r}^b$, after which further enhancements in explicitness do not strengthen the degree of anchoring. The same is true for the target's credibility.[15]

NOTES

1. See, for example, Beechey, Johannsen, and Levin (2008), Gürkaynak, Levin, and Swanson (2006), Gürkaynak, Sack, and Swanson (2005), Levin, Natalucci, and Piger (2004), or Kuttner and Posen (1999).

2. Both papers imply that the anchoring effect occurs under many—but not all—circumstances, i.e., explicit targets are not a sufficient condition for anchored expectations. By focusing specifically on the empirically relevant anchoring effect, they also leave room for a more a complete welfare assessment of explicit inflation targets (especially in light of the global financial crisis).

3. It will become apparent that the nature of our results is largely independent of the details of the macroeconomic model. For example, the intuition obtains for various forms of the shock process (including AR1) that have a zero mean.

4. Research shows that the central bank's relative weight on output in industrial countries has been fairly low; see, for example, Clarida, Gali, and Gertler (1998).

5. Most explicit inflation targets in industrial countries are specified as a long run objective, and interpreted in such a flexible fashion that allows for output stabilization.

6. The 1989 Reserve Bank of New Zealand Act serves as a real world example.

7. In Libich (2009) r^b is also endogenous. To keep the focus of the presented paper on the behavior of expectations, we will treat r^b as exogenous here.

8. Importantly, note that this punishment is the public's optimal choice, not an arbitrary rule (trigger strategy) of the Barro–Gordon variety.

9. Note that due to our asynchronicity restriction (6), $\hat{r}^p$ should be rounded to the nearest real value such that r^b/r^p is an integer. Nevertheless, because (6) is a purely technical assumption, in what follows we will use the more illustrative original condition in (10).

10. Note, however, that in many settings in which the public is *uncertain* about the value of x^T, the monitoring motive is likely to exist even under $x^T = 0$.

11. Let us point out that our game theoretic representation is quite general—it can nest any macroeconomic model, whereby the payoffs $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$ are simply functions of the deep parameters of the selected model.

12. Note that the proposition does *not* state that inflation never deviates from the inflation target. It is only in the long run (average) sense; see the discussion of Section 2.4.

13. The discussion of note 9 applies here as well [except that $\tilde{r}^p$ has to be rounded *down* because (17) is an inequality].

14. Standard game theoretic concepts do not offer a way to determine which of the multiple SPNE ends up being played. To simplify the presentation of the results, we will throughout assume that only the symmetric equilibria in (1) and (2) would be selected. Specifically, let us assume that (i) under $r^p = \tilde{r}^p = \frac{r^b}{2+\alpha}$ the Ramsey SPNE would obtain (for which a weak-dominance argument can be used) and (ii) under all the remaining values of the interval in (3) the anti-Ramsey SPNE would obtain.

15. Also note that if the public's monitoring motive exists but is insufficiently *strong*, i.e., $x^T \neq 0$ but $c_\pi < \tilde{c}_\pi$, then expectations may be inattentive but not anchored at the optimal inflation level.

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