# Incorporating Rigidity in the Timing Structure of Macroeconomic Games [1]

Jan Libich[2]
*La Trobe University, Dep. of Economics and Finance, and CAMA, ANU*

Petr Stehlik
*University of West Bohemia, Department of Mathematics*

Abstract

This paper proposes a simple framework that generalizes the timing structure of macroeconomic (as well as other) games. Building on alternating move games and models of 'rational inattention' the players' actions may be rigid, ie optimally chosen to be infrequent. This rigidity makes the game more dynamic/asynchronous and by linking successive periods it can serve as commitment. Therefore, it can enhance cooperation and often eliminate inefficient equilibrium outcomes. We apply the framework to the Kydland-Prescott-Barro-Gordon monetary policy game and derive the conditions - the sufficient degree of commitment - under which the influential time-inconsistency problem disappears. Interestingly, (i) this can happen even in a finite game (possibly as short as two periods), (ii) the required degree of commitment may be rather (even infinitesimally) low and (iii) the policymaker's commitment may substitute her conservatism and/or patience in achieving credibility. The analysis makes several predictions about explicit inflation targeting and central bank independence (and their relationship) that we show to be empirically supported. In doing so we show that our theoretical results reconcile some conflicting empirical findings of the literature.

**Keywords:** asynchronous moves, dynamic games, commitment, rigidity, time inconsistency, inflation targeting, central bank independence
**JEL classification:** C70, C72, E42, E61

'Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer... It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous. But at present, models of such realism seem beyond the power of our analytical tools'. Tobin (1982) quoted in Reis (2006).

## 1. Introduction

The macroeconomic theory has long taken notice of various rigidities seeking explanations for some observed phenomena. Empirical research followed and provided convincing micro-level evidence of the rigidity of, among other, prices and wages.[3] The presented paper takes rigidity a level up and incorporates it into the timing structure of macroeconomic games.

[1] We thank Damien Eldridge, Andrew Hughes Hallett, Jonathan Kearns, Glenn Otto, Jeff Sheen and the participants of the 23rd Australasian Economic Theory Workshop, 10th Australasian Macroeconomics Workshop, 34th Australian Conference of Economists, and seminars at the University of New South Wales, La Trobe University, University of Bayreuth, and University of Queensland. The usual disclaimer applies.

[2] Corresponding author: Jan Libich, La Trobe University, School of Business, Melbourne, Victoria, 3086, Australia. Email: j.libich@latrobe.edu.au.

[3] For recent surveys of empirical evidence see Apel, Friberg and Hallsten (2005) and Bewley (2002) respectively. For the seminal theoretical contributions see eg Fischer (1977), Taylor (1980), Calvo (1983), or Mankiw and Reis (2002).

The motivation is to bridge a gap between the micro-founded models of the economy used in macroeconomics, in which rigidities play a central role, and the rigidity-free solution concepts applied to these very models. This refers to both repeated games and the rational expectations solution − in both it is commonly assumed that players move simultaneously and do so each period. Since both the 'simultaneity' and the 'flexibility' assumptions have been questioned we provide an alternative framework to consider situations in which players' choices may be (optimally chosen to be) infrequent in the spirit of Tobin's quote.[4]

It will become apparent that as rigidity ties the hands of the players it (i) makes the environment more dynamic and asynchronous and (ii) takes the role of commitment.[5] This implies that it can help enhance cooperation between players in settings in which inefficient outcomes may otherwise result in equilibrium. Let us spell out some of the advantages of our framework:

**Generality.** The framework can be applied to any model in discrete time, continuous time as well as time scales (the letter being a recent generalized mathematical environment which nests both discrete and continuous time as special cases, see eg Bohner and Peterson (2001)). Further, unlike a standard repeated game our framework enables us to examine:
(i) concurrent rigidity/commitment of more than one player
(ii) various degrees of such rigidity/commitment
(iii) endogenous determination of rigidity/commitment as players optimal choices.

Some of these features (one at a time) have already been examined in games.[6] This existing work provides a strong justification and motivation for our general approach; for example Cho and Matsui (2005). argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes. . . '.*

**Familiarity.** This paper focuses on the standard discrete (constant) time used in most macroeconomic models.[7] Two main forms of discrete rigidity have been used in the literature - the Taylor (1979) deterministic and the Calvo (1983) probabilistic schemes. Our general setup in discrete time nests both forms and can be summarized by one parameter, $\theta_t^{i,m} \in [0,1]$, which denotes the probability that player $i$'s instrument $m$ cannot be altered in period $t$. Despite this extension, the framework adopts all the main assumptions of a standard repeated game, eg it starts with a simultaneous move, rigidity/commitment is constant throughout each game, and all past periods' actions are observable (ie games of 'almost perfect information').

**Realism.** It will be clear that players' rigidity and commitment introduce some asynchronicity in the game and make the game more dynamic. This combination of perfect and imperfect information is arguably a good description of many repeated real world interactions. Further,

---

[4]In terms of simultaneity, Lagunoff and Matsui (1997) argue that *'[w]hile the synchronized move is not an unreasonable model of repetition in certain settings, it is not clear why it should necessarily be the benchmark setting. . . '.* In terms of incorporating some inflexibility, in addition to the above see a growing body of literature examines some sort of inertia/stickiness/rigidity in updating/forming expectations (see eg Ball (2000), Mankiw and Reis (2002), Carroll (2003), Carroll and Slacalek (2006), Morris and Shin (2006)). This is further consistent with the concept of 'economically rational expectations' (Feige and Pearce (1976) as well as with 'rational inattention' (Sims (2003), Reis (2006)) - in which updating expectations is a result of a cost/benefit analysis by the agents.

[5]The terms rigidity and commitment can be used interchangeably in our framework. While a game theorist will find it natural to think of commitment (since the interest lies in the effect on the game), a macroeconomist may want to use the term that better reflects the particular underlying circumstances. We will follow the latter practice in this paper as it is targeted at the macroeconomics audience.

[6]Feature (i) is investigated in the alternating move games of Maskin and Tirole (1988), Lagunoff and Matsui (1997) and Cho and Matsui (2005). In terms of (ii) Wen (2002) examines a first simple step in this direction. Finally, Bhaskar's (2002) work in which leadership is endogenously determined offers an avenue to incorporate (iii).

[7]For detailed treatment of continuous and time scales calculi see Libich and Stehlik (2006) - these will be outlined in section 8.

the framework does not rely on the infinite horizon - a unique equilibrium (the efficient one) can commonly be obtained in a finite game even without reputational considerations. Finally, the framework captures Tobin's observation quoted above about varying frequency of agents' actions and its endogeneity.

**Simplicity.** While allowing for the above extensions some general results will be proven that demonstrate the tractability of our framework. In most settings the game can be solved by subgame perfection. Furthermore, the solution is often as simple as that of a one shot game since the most important 'action' will occur in the initial simultaneous move (Theorem 2).

To demonstrate the framework we use one of the most influential macroeconomic games due to Kydland and Prescott (1977) and Barro and Gordon (1983) (referred to as BG).[8] It is shown under what circumstances the famous time inconsistency result is qualified in the rigid world. Specifically, we derive the sufficient conditions for the efficient 'Ramsey' outcome of credibly low inflation - that is not a Nash equilibrium in the standard 'rigidity-free' game - uniquely obtains in equilibrium of the rigid game (on the equilibrium path of all subgame perfect Nash equilibria, Propositions 1-2 and Theorem 3). It is further shown that the required degree of commitment is a function of the characteristics of the economy and the players' preferences (Proposition 3); and, interestingly, that it can be infinitesimally low (Theorem 1).

The main policy result is that monetary commitment can substitute central bank goal-independence (conservatism and/or patience) in ensuring the credibility of low inflation (Proposition 4). We discuss how the policymaker's commitment has been achieved in the real world context drawing a link to the observed trend towards explicit inflation targeting and transparency. Since the inflation target (IT) is transparently incorporated in the central banking legislation, it cannot be frequently reconsidered and the choice of the *long-run* inflation level is therefore rigid.[9]

This substitutability offers an explanation for the fact that ITs have been made more explicit in countries that had lacked central bank goal-independence in the late 1980s such as New Zealand, Canada, UK, and Australia rather than those with an independent central bank such as the US, Germany and Switzerland.[10]

The rest of the paper is structured as follows. Section 2 introduces our general rigid framework in discrete time and outlines various cases of interest. Section 3 presents two versions of the BG game, one general and one specific. Section 4 sets up the scene of the deterministic setup that is the focus of this paper. Sections 5 and 6 report results under the players' patience and impatience respectively - some of which extend to other classes of games as well. Section 7 brings empirical evidence for our results, also reconciling some contradictory findings of the existing literature. Section 8 discusses their robustness and a number of extensions. Section 9 summarizes and concludes.

## 2. Introducing Rigidity/Commitment: Discrete Time

In the general discrete framework time is denoted by $t \in \mathbb{N}$. There are $I$ players who each have $M^i$ instruments (choice variables). Each instrument $m \in M = \cup M^i$ can take $l^{i,m}$ levels. Denote the probability that player $i \in I$ cannot move instrument $m \in M$ in period $t$ by $\theta_t^{i,m}$. Naturally, we assume that

$$0 \leq \theta_t^{i,m} \leq 1.$$

[8]In Hughes Hallett and Libich (2006) a different macroeconomic game - the monetary-fiscal policy interaction (game of chicken) is examined.

[9]Based on the analysis in Libich (2006b) it will be argued below that this does not reduce the policymaker's *short run* flexibility to stabilize shocks and output.

[10]For the lively inflation targeting debate in regards to the Fed see Bernanke (2003), Goodfriend (2003), Kohn (2003), McCallum (2003), Friedman (2004), Mishkin (2004), or Lacker (2005).

This nests the standard repeated game (in which $\theta_t^i = 0$, $\forall i, t$) as well as the alternating move game (in which the respective probabilities for the two players $i$ and $j$ are, $\forall t$, $\theta_t^i = \frac{(-1)^t+1}{2}$ and $\theta_t^j = \frac{(-1)^{t+1}+1}{2}$). The most natural discrete cases to examine are the two common specifications of rigidity, namely the Taylor (1979) deterministic and the Calvo (1983) probabilistic schemes and their combinations.

1) Purely deterministic: $\theta_t^i = 0, \forall i$ and $\forall t = 1 + (n-1)r^i$ where $n \in \mathbb{N}, r^i \in \mathbb{N}, \forall i$, otherwise $\theta_t^i = 1, \forall i$.

2) Purely probabilistic: $\theta_t^i = \theta, \forall i, t$, and some $0 \leq \theta \leq 1$.

3) Combination *within* players: $\theta_t^i = 0, \forall i$ and $\forall t = 1 + (n-1)r^i$ otherwise $\theta_t^i = \theta, \forall i$.

4) Combination *across* players: $\forall j \neq i, \theta_t^j = 0, \forall t = 1+(n-1)r^j$ otherwise $\theta_t^j = 1$. In contrast, $\theta_t^i = \theta, \forall t$.[11]

In this paper we concentrate on the first setup.[12] It will become clear below that not only is this intuitive case easiest to analyze but it communicates the essence and richness of asynchronous decision making as well. Furthermore, it best fits Tobin's observation mentioned above about varying frequencies of agents' actions.

## 3. A stylized Macroeconomic Game

Since the BG game models the interactions between two players who each have one instrument we will focus on this special case, $I = \{p, g\}, M^p = M^g = 1$. Further, for game theoretic clarity we will restrict the players' action set to two levels, $l^{i,m} = 2, \forall i, m$, and specifically a low ($L$) and a high ($H$) level, $l^p = l^g = \{L, H\}$.

### 3.1. General BG Game.

We can interpret $p$ as the public and $g$ as the policymaker. In its general form the time-inconsistency game can be summarized by the payoff matrix in Figure 1.

| | | *Public* | |
|---|---|---|---|
| | | $L$ | $H$ |
| *Policymaker* | $L$ | $a,\ q$ | $b,\ v$ |
| | $H$ | $c,\ x$ | $d,\ z$ |

Figure 1. General BG game payoffs

The real parameters $a, b, c, d, q, v, x, z$ denote the players' payoffs that satisfy the following *general* conditions

$$c > a > d > b,\ \ q > v,\ \text{and}\ q \geq z > x. \tag{1}$$

We will refer to $a$ as the credibility benefit, $-b$ as the disinflation cost, $c$ as temptation, $-d$ as the inflation cost. The $(H, H)$ outcome is the unique Nash equilibrium of the game, however, it is Pareto inferior to the $(L, L)$ outcome.

[11]Note that it may be desirable to qualify the probabilistic cases 2 and 4 to start with a simultanoues move with *certainty*, ie $\theta_1^i = 0, \forall i$, to uniquely determine the play in period 1.

[12]Libich and Stehlik (2006) examine setups 2-4.

3.2. **Specific BG game.** The most common way to describe the economy in this setting is using a simple Lucas surprise-supply relationship

$$y_t - Y = \lambda(\pi_t - w_t) + \varepsilon_t, \tag{2}$$

where $\lambda > 0$, $y$ denotes output, $Y$ denotes the natural output level, and $\varepsilon$ is a cost push shock with zero mean. The players' discount factors are $\delta_g$ and $\delta_p$ and their one period utility functions are the following

$$u_t^g = -(\pi_t - \tilde{\pi})^2 + \alpha y_t - \beta(y_t - Y)^2, \tag{3}$$

$$u_t^p = -(\pi_t - w_t)^2, \tag{4}$$

where $\tilde{\pi}$ is the socially optimal inflation level (explicit or implicit target), and $\alpha > 0, \beta \geq 0$ describe the policymaker's relative weight between its objectives (of stable inflation, high output, and stable output). The policymaker's and the public's instruments are $\pi$ and $w$ respectively, $m \in \{\pi, w\}$. The intuition is standard, the public cares about correctly expecting the inflation rate in order to set wages at the market clearing real wage level (for a justification based on Fischer-Gray contracts that is in line with our setting see Canzoneri (1985)). This is equivalent to 'rational expectations' in a rigidity-free repeated game.[13]

**Long-run Perspective.** Since our interest lies in the effect of policy commitment we will focus on long-run/average/trend outcomes of the game. To do so we will make the economy deterministic by setting $\varepsilon_t = 0, \forall t$, which implies that we can set $\beta = 0$ without loss of generality. It then follows that the policymaker's instrument $\pi$ represents choosing *average* inflation or a certain level of a *long-run IT*.[14]

In the standard repeated game in which players can alter their actions every period, ie $\theta_t^i = 0, \forall t, i \in I$, we use (2)-(3) to obtain the equilibrium levels (denoted by star throughout)

$$\pi_t^* = \tilde{\pi} + \frac{\alpha\lambda}{2} = w_t^*. \tag{5}$$

This is the famous inflation bias result, $\pi_t^* > \tilde{\pi}$. In restricting our attention to two action levels we follow Cho and Matsui (2006) who depict the most natural candidates - the optimal level from (3) and the time-consistent level from (5)

$$\pi \in \{L = \tilde{\pi}, H = \tilde{\pi} + \frac{\alpha\lambda}{2}\} \ni w.$$

We can, taking into account (2)-(4) and dividing through by $(\frac{\alpha\lambda}{2})^2$ without loss of generality, derive the respective payoffs (reported in Figure 2).

Regardless of $\lambda$ and $\alpha$ (ie for any policy weight), the following specific BG game constraints, in addition to the general ones in (1) are satisfied:

$$a = 0, c = -d = -\frac{b}{2} \text{ and } q = z, v = x. \tag{6}$$

[13]While in a rigidity-free environment the terms wage inflation and expected inflation can be used interchangeably, see eg Backus and Driffill (1985), it will become apparent that in the presence of wage rigidity these two differ. Libich (2006) uses adds expected inflation in (4) and shows that the findings are unchanged

[14]Long-run IT means that the legislated horizon of the target is the business cycle or longer (indefinite) - as is common in industrial countries, see Mishkin and Schmidt-Hebbel (2001). Since shocks have a zero mean they do not affect the average/trend levels - this is shown in the stochastic extension of this paper, Libich (2006b), which will be discussed in detail in section 8.

| | | *Public* | |
|---|---|---|---|
| | | $w^L$ | $w^H$ |
| *Policymaker* | $\pi^L$ | 0, 0 | -1, -1 |
| | $\pi^H$ | ½, -1 | -½, 0 |

Figure 2. Specific BG game payoffs

In the rest of the paper *specific BG game* will refer to a game in which both (1) and (6) hold (but the exact payoffs of Figure 2 do not necessarily apply). Note therefore that our specific BG game still features a fair amount of generality.

Our aim is to investigate under what conditions the time inconsistency and inflation bias results disappear in the rigid setting. While the effect of the policymaker's commitment is well known our innovation is in deriving the exact degree of commitment under which this will be achieved and under which the optimal low inflation level/target becomes credible. By *credibility* of low inflation we will mean a situation of $\pi^L$ accompanied by $w^L$ - whereas if it is accompanied by $w^H$ we will talk about 'lack of credibility'.

## 4. Deterministic Setup

4.1. **Assumptions.** We adopt all the assumptions of a standard repeated game - a number of alternative specifications are discussed in section 8. First, rigidity/commitment are discrete and constant throughout each game. Second, they are common knowledge. Third, all past periods' moves can be observed. Fourth, the game starts with a simultaneous move with certainty which may be interpreted as reflecting some 'initial' uncertainty. Fifth, players are rational, have common knowledge of rationality and for expositional clarity they have complete information about the structure of the game and opponents' payoffs.

**Definition 1.** *Player $i$'s instrument $m$'s* ***deterministic rigidity (commitment)****, $r^i_m \in \mathbb{N}$, expresses the number of periods for which the respective action cannot be altered.*

**Definition 2.** *An* ***unrepeated asynchronous game with deterministic rigidity (commitment)*** *is an extensive game that starts with a simultaneous move of all $m \in M$, continues with moves every $r^i_m$ periods, and finishes after $T$ periods, where $T \in \mathbb{N}$ denotes the 'least common multiple' of $r^i_m, \forall i, m$.*

Since in our BG game we have $M^i = 1, \forall i$, we will drop the subscript $m$. An example of such game in the form of a time line is presented in Figure 3.

We will interpret $r^p$ as *wage rigidity* following Taylor (1979) and $r^g$ as the strength of the monetary policy *long-run commitment*. From the fact that $\pi$ represents setting a certain level of a long-run IT it follows that $r^g$ can be interpreted as the degree of the *target's explicitness*. This is because the more explicitly the IT is stated in the central banking legislation the less frequently it can be altered (in the Taylor (1979) deterministic sense) or the less likely it is (in the Calvo (1983) probabilistic sense). As a real world example of deterministic $r^g$ the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a

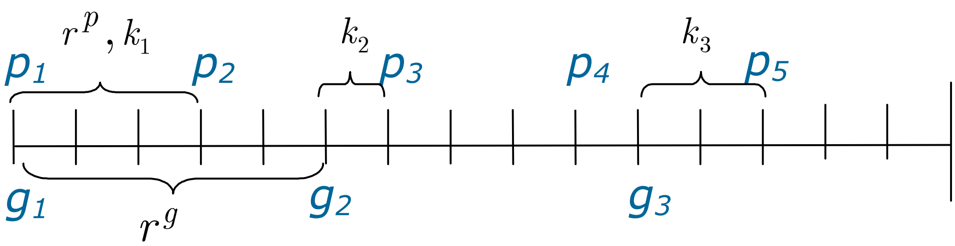



FIGURE 3. Unrepeated asynchronous game with determinictic discrete rigidity and commitment - an example of timing of moves.with $r^g = 5$ and $r^p = 3$ ($k$ will be defined below).

Policy Target Agreement between the Minister of Finance and the Governor and this can only be done on *pre-specified regular* occasions (eg when a new Governor is appointed).[15]

While this asynchronous game can be repeated we can restrict our attention to the unrepeated game (as depicted in Figure 3). This is because we will be deriving conditions under which the efficient outcome uniquely obtains in equilibrium of the unrepeated game. If these conditions are satisfied repeating the game and allowing for reputation building of some form would not affect the derived equilibrium.[16]

4.2. **Notation.** We denote the number of the $i$'s player's move by $n^i$ and the number of his final move in the unrepeated game by $N^i$. It then follows that $N^i = \frac{T(r^g, r^p)}{r^i}$. Also, $g_n^l$ and $p_n^l$ will denote a certain action $l \in \{L, H\}$ in a certain node $n^i$, eg $p_2^H$ refers to the public's high wage play in its second move. Let us introduce the notation for the case of interest $r^g \geq r^p$. Denote $\frac{r^g}{r^p} \geq 1$ to be the players' relative rigidity. Further, $\lfloor \frac{r^g}{r^p} \rfloor \in \mathbb{N}$ will be the integer value of relative rigidity (the floor) and $R = \frac{r^g}{r^p} - \lfloor \frac{r^g}{r^p} \rfloor = [0, 1)$ denotes the fractional value of relative rigidity (the remainder).[17] Further, we denote $b(.)$ to be the best response. For example, $p_1^L \in b(g_1^L)$ expresses that $w^L$ is the public's best response to the policymaker's initial $L$ move.[18] Let us also repeat here that a star denotes optimal play, ie $p_1^* \in b(g_1)$ expresses that the public's optimal play in move 1 is the best response to the policymaker's first move (regardless of $l^g$). Finally, threshold levels will be denoted by either upper bar (for sufficient ones) or hat (for necessary *and* sufficent ones).

4.3. **Recursive Scheme.** Our proofs are based on the recursive scheme implied by our setup and number theory. Let us use $k_n$ to denote the number of periods between the $n^g$-th move of the policymaker and the immediately following move of the public (which implies $k_1 = r^p$, see Figure 3). From this it follows that the number of periods between the $(n^g + 1)$-th move of the

[15]It should further be noted that the absence of a legislated numerical target may not necessarily imply $r^g = 1$; it has been argued that many countries pursue an inflation target implicitly (including the US, see e.g. Goodfriend (2003), or the Bundesbank and the Swiss National Bank in the 1980-90s, see Bernanke, et al. (1999)). In such cases we have $r^g > 1$.

[16]In this sense we can think of our analysis as the worst case scenario in which reputation cannot help cooperation.

[17]It will be evident that $R$ plays an important role since it determines the exact type of dynamics (asynchronicity) in the game.

[18]Note however, that from the game theoretic definition of best response this best response may not be unique (ie $p_1^L \in b(g_1^L)$ is not equivalent to $b(g_1^L) = \{p_1^L\}$ - as it may still be true that $p_1^H \in b(g_1^L)$).

policymaker and the immediately preceding move of the public equals $r^p - k_{n+1}$. Using these we can summarize the recursive scheme of the game as follows:

$$k_{n+1} = \begin{cases} k_n - Rr^p & \text{if} \quad k_n \geq Rr^p, \\ k_n + (1-R)r^p & \text{if} \quad k_n < Rr^p, \end{cases} \tag{7}$$

Generally, $k_n$ is not a monotone sequence, see eg Figure 3.

4.4. **History and Future.** By convention, history in period $t$, $h_t$, is the sequence of actions selected prior to period $t$ and future in period $t$ is the sequence of current and future actions. It follows from our 'observability' assumption that $h_t$ is common knowledge at $t$. Let us introduce the concept of 'recent' history that will span from the last move of the opponent to the present period $t$.[19] From (3) and (4) (in which the payoffs in period $t$ is only a function of the opponent's action in $t$) and the recursive mechanism in (7) it follows that anything prior to the recent history never affects the players' play. Furthermore, we will see that even recent history has no affect on some actions - we will refer to such actions as 'history-independent'.

4.5. **Strategies and Equilibria.** A strategy for a certain player is a function that, $\forall h_t, t$, assigns a probability distribution to the player's action space. As common in macroeconomics, in this paper we will restrict our attention to pure strategies. A strategy of player $i$ is then a vector that, $\forall h_t$, specifies the player's play $\forall n^i$.

The asynchronous game will commonly have multiple Nash equilibria. To select among these we will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. Subgame perfect Nash equilibrium (SPNE) is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history $h_t$.[20]

Given the large number of nodes in the game reporting fully characterized SPNE would be cumbersome. We will therefore focus on the *equilibrium path* of the SPNE, ie actions that will actually get played.[21] To simplify the language let us define the following.

**Definition 3.** *Any SPNE in which both players play L in all their moves on the equilibrium path, $(i_n^L)^*, \forall n, i$, will be called* ***Ramsey****.*

4.6. **Discounting.** To make the exposition more illustrative we will first examine the game under the assumption of (fully) *patient* players, $\delta_i = 1, \forall i$, and then consider the effect of the players' *impatience,* $\delta_i < 1$. As the intuition of the rigid environment is independent of the players' discount factor most of the results will carry over. It will be shown that while the public's impatience improves cooperation the policymaker's impatience has the opposite effect.

## 5. Results: Fully Patient Players

For the sake of transparency we first focus on the results of our specific BG game, in which both (1) and (6) are satisfied. Then we extend them (using the derivations of the specific BG game) to the general setup where only (1) is required to hold.

[19]For example, in terms of the policymaker's $n^g > 1$ move in period $t$ *it* follows from using (7) that recent history starts in period $t - r^p + k_n$.

[20]Note that the specification of the players' utility implies that all our SPNE will also be Markov perfect equilibria, for details see eg Maskin and Tirole (2001).

[21]To demonstrate, for our example in Figure 3 each SPNE consists of $\sum_{s=1}^{r^p} \sum_{f=1}^{r^g} 2^{(s+f-1)} = 254$ actions whereas on its equilibrium paths there are $r^p + r^g = 8$ actions.

**Proposition 1.** *Consider the specific BG game in which* (1) *and* (6) *hold, and assume* $\delta_p = \delta_g = 1$ *and*

$$\frac{r^g}{r^p} \in \left(\frac{3}{2}, 2\right) \cup \left(\frac{5}{2}, \infty\right). \tag{8}$$

*Then any SPNE of the game is Ramsey.*

*Proof.* We solve the game backwards and prove the statement by a mathematical induction argument with respect to the policymaker's moves. First, we prove that on the equilibrium path $L$ will be played in the policymaker's last move $n^g = N^g$ (the inductive basis). Then, supposing that it holds for some $n^g \leq N^g$, we show that the same is true for $(n^g - 1)$ as well. This will prove that on the equilibrium path we have $g_n^L, \forall n^g$. Since the public's unique best response to $L$ is $L$, it will follow that in equilibrium $p_n^L, \forall n^p$.[22]

$\mathbf{n^g = N^g}$ **under** $\mathbf{R = 0}$: Focusing first on this special case is illustrative. Here we have $T(r^g, r^p) = r^g$ and therefore $N^g = 1$ (and $N^p = r^g$). For there to exist a Ramsey SPNE it is therefore required that $g_1^L \in b(p_1^L)$. This condition will also ensure $\pi^L$ to be time consistent and possibly (but not surely) credible.[23]

Solving backwards, we know that $p_{n>1}^* \in b(g_1)$ due to perfect information in $n^p > 1$. Further, from the public's rationality and complete information it follows that $p_1^* \in b(g_1)$. Using this yields the following condition

$$ar^g \geq cr^p + d(r^g - r^p). \tag{9}$$

The left-hand side (LHS) and the right-hand side (RHS) will throughout report the policymaker's payoffs from playing, in a certain node, $\pi^L$ and $\pi^H$ respectively. Since (9) assumes $p_1^L$ then if the policymaker inflates (ie $g_1^H$) he manages to surprise the public and gets a boost in output, $c$. This however only lasts for $r^p$ periods after which the public would switch to $w^H$ and 'punish' the policymaker with a $d$ payoff for the rest of the unrepeated game.[24] Intuitively, for $\pi^L$ to be time consistent the inflation cost has to offset temptation.

For any SPNE of the game to be Ramsey it is, in addition to (9), required that, $\forall n^g$, $b(w^H) = \{g_n^L\}$ which yields the following condition

$$br^p + a(r^g - r^p) > dr^g. \tag{10}$$

If the condition is satisfied $\pi^L$ is played regardless of the initial public's move and is therefore surely credible.

**Lemma 1.** *In the specific BG game in which* (1) *and* (6) *hold the conditions* (9) *and* (10) *are equivalent (except for weak/strict inequality).*

*Proof.* Using $a = 0$ and $c = -d = -\frac{b}{2}$ from (6), we can transform (9) into

$$0 \geq dr^g - br^p,$$

and consequently, $a = 0$ implies (10). □

**Remark 1.** *It is straightforward to show that the equivalence of Lemma 1 holds, due to the specific BG game constraints in* (6), $\forall n^g, R$.[25]

[22] It will become evident that for most parameter values satisfying (8) there will be a unique Ramsey SPNE but since our attention will be on the equilibrium path we will not examine the number of SPNE (off-equilibrium path behaviour) in detail.

[23] Credibility may lack since $w^H$ may still by the public's optimal play.

[24] Note that unlike in Barro and Gordon (1983a, 1983b), the punishment in the rigid world is not arbitrary but it is the public's optimal play and its length is uniquely determined by wage rigidity.

[25] In fact the same holds $\forall \delta_g, \delta_p$ as well.

Therefore, in the rest of the proof of Proposition 1 we can focus our attention only on equation (10) and analogous ones that ensure $b(w^H) = \{g_n^L\}$ at every $n^g$. Rearranging (10) yields

$$r^g > \frac{a-b}{a-d} r^p \overset{(6)}{=} 2r^p, \tag{11}$$

where the first element of the RHS only uses the general payoff constraints in (1) whereas the second uses the specific ones from (6). Therefore, under $R = 0$ all $\frac{r^g}{r^p} = 2, 3, 4, \ldots$ deliver a (unique) Ramsey SPNE but in the case of $\frac{r^g}{r^p} = 2$ there also exists one SPNE that is not Ramsey (specifically, one in which $i_n^H, \forall n, i$).

**$\mathbf{n^g = N^g}$ under $\mathbf{R = (0, 1]}$:** From Definition 2 it follows that the number of the policymaker's moves in the unrepeated game is $N^g = \frac{T(r^g, r^p)}{r^g}$. A condition analogous to (10) is the following

$$br^p R + a(r^g - r^p R) > dr^g. \tag{12}$$

Rearrange this to obtain

$$r^g > \frac{a-b}{a-d} R r^p, \tag{13}$$

which is, due to $R < 1$, weaker than (10) for all constraints satisfying (1). This means that, if (11) holds, a patient policymaker will find it optimal to play $g_N^L$ for all histories.

**$\mathbf{n^g + 1 \to n^g}$** (if applicable, ie if $1 \le n^g < N^g$): We assume that the policymaker's unique best play in the $(n^g + 1)$-th step is $L$ regardless of the public's preceding play (ie that $g_{n+1}$ is history-independent), and we attempt to prove that this implies the same assertion for the $n^g$-th step. Intuitively, this means that if the policymaker inflates he finds it optimal to immediately disinflate. Two scenarios are possible - the disinflation will either be costly - lacking credibility (due to excessive wages $w^H$ the payoff $b$ occurs for at least one period) or costless (only accompanied by $w^L$ and the payoff $a$).

Whether the disinflation is costly or costless depends on the recursive scheme in (7) and the public's preferences. As discussed in section 4.4 the public's optimal play in any $n^p > 1$ is never affected by play prior to the recent history. In particular, the public plays the best response to either the immediately preceding or the immediately following move of the policymaker (note that in the latter case the action is history-independent). This implies that there are two conditions, analogous to (10), that may apply in a certain $n^g$. The costly disinflation condition for all relevant nodes of the policymaker - derived in the same way as (10) - is the following

$$bk_n + a(r^g - k_n) + a[r^p - (r^p - k_{n+1})] > dr^g + b[r^p - (r^p - k_{n+1})], \tag{14}$$

where the last term expresses the total cost of disinflation. Similarly the costless disinflation condition will be

$$bk_n + a(r^g - k_n) > d[r^g - (r^p - k_{n+1})] + c(r^p - k_{n+1}), \tag{15}$$

where the last component expresses the output gain from the public's switch prior to the disinflation. Which of these two conditions is relevant to a certain $n^g$ depends on the public's payoffs $q, v, x, z$, and importantly on $k_{n+1}$. Specifically, if

$$(r^p - k_{n+1})z + k_{n+1}w \ge (r^p - k_{n+1})x + k_{n+1}v, \tag{16}$$

then (14) obtains otherwise (15) is the relevant condition.[26]

---

[26] For example in the specific BG game (in which $q = z, x = v$ from (6)) with patient public, $\delta_p = 1$, the disinflation will be costly if $k_{n+1} \le r^p/2$ and costless if $k_{n+1} > r^p/2$. Using our game in Figure 3 if $g_1^H$ then the disinflation in the next move, $g_2^L$, is costly since the public's unique optimal play is $p_2^H$ (the best response to $g_1$). In contrast, if $g_2^H$ then the disinflation in $g_3^L$ is costless since the public's optimal play is $p_4^L$ (the best response to $g_3$, not $g_2$). We will see in section 6.1 that the parameter space under which (15) obtains is getting smaller with the public's impatience.

Now, we will show for the specific BG game that if the conditions (14) and (15) are satisfied at $n^g = 1$, then they hold in all other $n^g$ as well.

**Lemma 2.** *Consider the specific BG game in which* (1) *and* (6) *hold. Then for all R the sufficient condition to ensure the only Ramsey SPNE relates to* $n^g = 1$ *(the initial simultaneous move).*

*Proof.* Equations (14) and (15), respectively, can be rearranged into

$$r^g > \frac{a-b}{a-d}(k_n - k_{n+1}), \tag{17}$$

$$r^g > \frac{(a-b)k_n + (c-d)(r^p - k_{n+1})}{a-d}. \tag{18}$$

In the case of the former inequality the strongest condition is guaranteed by the maximum of the difference $(k_n - k_{n+1})$. From (7) it follows that $k_n - k_{n+1} \leq Rr^p$. Since $k_1 - k_2 = Rr^p$, the statement is implied for all values of the *general* BG game. In terms of (18) it is not the case. To see this note that $k_{n+1}$ is a function of $k_n$ and substitute away from (7) - given that the strength of the condition is decreasing in $k_{n+1}$ we need to use $k_{n+1} = k_n - Rr^p$. Then (18) becomes

$$r^g > \frac{(a-b-c+d)k_n + (c-d)(1-R)r^p}{a-d}.$$

It follows that the RHS is non-decreasing in $k_n$ (ie the condition is sufficient at $n^g = 1$) if $a - b \geq c - d$. This does include our specific BG game constraints in (6). □

Continuing the proof of Proposition 1 and using $k_1 = r^p$ and $k_{n+1} = k_n - Rr^p$ jointly yields $k_2 = (1-R)r^p$. Substituting these into (17) and (18) we obtain

$$r^g > \frac{a-b}{a-d}Rr^p \overset{(6)}{=} 2Rr^p \quad \text{if} \quad R \geq 0.5, \tag{19}$$

$$r^g > \frac{a-b+R(c-d)}{a-d}r^p \overset{(6)}{=} 2(1+R)r^p \quad \text{if} \quad R < 0.5. \tag{20}$$

The intervals in terms of $R$ in (19) and (20) follow from the intervals for a costly disinflation ($k_{n+1} \leq r^p/2$) and a costless one ($k_{n+1} > r^p/2$) (see footnote 5) combined with $k_2 = (1-R)r^p$. This in conjunction with (19) and (20) completes the proof of Proposition 1.[27] □

Having built up the intuition of the asynchronous deterministic framework we can now extend this result to the general case.

**Proposition 2.** *Consider the general BG game in which* (1) *holds and* $\delta_g = \delta_p = 1$. *For every* $r^p$ *there exists a sufficient commitment level,* $\overline{r^g} \in \mathbb{N}$, *such that for all* $r^g > \overline{r^g}$ *any SPNE of the game is Ramsey.*

*Proof.* In order to extend Proposition 1 it suffices to circumvent the parts of Lemma 1 and Lemma 2 in which the special constraints (6) have been exploited. It is straightforward to see that the conditions for $\pi^L$ to be played regardless of the history $\forall n^g, R$ (relating to both $b(w^L) = \{g^L_n\}$ and $b(w^H) = \{g^L_n\}$) can be rewritten in the following form

$$(a-d)r^g > K, \tag{21}$$

[27] It is illustrative to consider why some low commitment values (in particular in the intervals $\frac{r^g}{r^p} \in [1, 1.5) \cup (2, 2.5)$ fail to deliver the Ramsey equilibrium in the specific BG game. It is because the relative length of the public's punishment is insufficient to discourage the policymaker from inflating. For example under $r^g = 4, r^p = 3$ there would be no punishment whatsoever (the play on the equilibrium path is $g^H_1, p^H_1, p^L_{n=\{2,3,4\}}, g^L_{n=\{2,3\}}$). It is shown below that this 'forgiving' behaviour disappears if the public is 'sufficiently impatient' or has adaptive expectations.

where $K$ is finite as it is a function of $r^p, \forall R$, and of some/all of $a, b, c, d$ depending on $R$. This follows directly from the fact that $b(\pi_t^L) = \{w_t^L\}$ (yielding the payoff $a$ on the LHS of each condition such as (9), (10), (14) or (15)) and $b(\pi_t^H) = \{w_t^H\}$ (yielding the payoff $d$ on the RHS of each such condition). Using the general constraint $a > d$ from (1) and the fact that $\overline{r^g}$ is finite proves the claim. □

In concluding this section, we summarize the dependence of commitment on wage rigidity and the parameters of the BG game.

**Proposition 3.** *Consider the general BG game in which* (1) *holds and assume* $\delta_p = \delta_g = 1$*. Then the degree of commitment that suffices to achieve only Ramsey SPNE,* $\overline{r^g}$*, is positively related to wage rigidity,* $r^p$*, the policymaker's temptation,* $c$*, and the disinflation cost,* $-b$*, and negatively related to the credibility benefit,* $a$*, and the inflation cost,* $-d$*.*

*Proof.* The proof of Lemma 2 shows that equations (17) and (18) yield the strongest conditions under maximal $k_n$ and minimal $k_{n+1}$ also for the general case. To derive the sufficient condition we will therefore examine the worst possible case $k_n = r^p$ and $k_{n+1} = 0$. Using these two extremal values we can rewrite (17) and (18) into

$$r^g > \overline{r^g} = \frac{a-b}{a-d} r^p, \tag{22}$$

and

$$r^g > \overline{r^g} = \frac{a-b+c-d}{a-d} r^p. \tag{23}$$

Since (23) is, for all general values of $a, b, c, d$, stronger than both (22) and (10), it is the sufficient condition of the general BG game. It can be rewritten as

$$r^g > \overline{r^g} = 1 + \frac{c-b}{a-d} r^p,$$

which, together with $r^p > 0$, $c > b$, $a > d$, proves the statement of the proposition. □

Arguably, the payoffs $a, b, c, d$ in the real world depend on various factors such as the structure of the economy, Union power, the way agents form expectations, political economy factors (lobby groups, political cycles), institutional setting of monetary policy etc.

## 6. Results: Impatient Players

In this section we consider more general settings in which the players discount future. To separate the effects of the public's and the policymaker's discounting we examine them in turns.

### 6.1. **The Public's Impatience.** 

This subsection shows that the public's discounting may *weaken* the above sufficient conditions for Ramsey SPNE and hence *improve* cooperation. The following result is a general finding that section 8 shows to apply to other classes of games as well - under some circumstances even an infinitesimal amount of (relative) commitment is sufficient to achieve the efficient outcome.

**Theorem 1.** *Consider the general BG game in which* (1) *holds and assume* $\delta_g = 1$, $0 \leq \delta_p \leq \overline{\delta_p} < 1$ *where* $\overline{\delta_p}$ *is some upper bound, and* $a \geq 2d - b$*. Then for all*

$$\frac{r^g}{r^p} \in (1, \infty),$$

*there exists a Ramsey SPNE.*

*Proof.* We claim that for some parameter values (including the specific BG game) any $r^g > r^p$ is sufficient. Under $R = 0$ the value of $\delta_p$ does not affect the relevant sufficient condition in (11). However, if $R = (0, 1]$ and the public is sufficiently impatient, $\delta_p \le \overline{\delta_p}$, where the threshold value $\overline{\delta_p}$ is a function of $r^g, r^p, q, v, x, z$, the sufficient condition will alter. Instead of deriving analytically $\overline{\delta_p}$ from (16) we focus on the extreme case $\overline{\delta_p} = \delta_p = 0$ which is a sufficiently low threshold $\forall r^g, r^p$ and $\forall q, v, x, z$ satisfying (1).

The impatient public will disregard the future and always play $w_t^* \in b(\pi_t)$. Since for all but the initial move the players never move simultaneously this implies $p^*_{n>1} \in b(\pi_{t-1})$.[28] Intuitively, a sufficiently impatient public will never reduce wages before the start of the disinflation, ie it will always punish inflating and make disinflation costly. Formally, (15) no longer applies and (14) becomes the relevant condition $\forall n^g, R = (0, 1]$, and $\forall a, b, c, d, q, v, x, z$ satisfying (1).[29]

Hence we need to show that any $r^g > r^p$ satisfy the following two conditions: (i) under $R = 0$ it holds that $\frac{r^g}{r^p} \ge \frac{a-b}{a-d}$ (from (11)) and (ii) $\forall R = (0, 1]$ it is true that $\frac{r^g}{r^p} \ge \frac{a-b}{a-d}R$ (from (19)). In terms of (ii) realize that any $r^g > r^p$ have, from the definition of $R$, the property that $\frac{r^g}{r^p} \ge 1 + R$. This implies that claim (ii) can be rewritten as $1 + R \ge \frac{a-b}{a-d}R$. Divide both sides by $R$ to obtain $\frac{1}{R} + 1 \ge \frac{a-b}{a-d}$. To see that this is satisfied utilize two characteristics. First, $\frac{1}{R} + 1 > 2$ since $R < 1$. Second, rearrange $a \ge 2d - b$ into $2 \ge \frac{a-b}{a-d}$. Combining these gives $\frac{1}{R} + 1 > 2 \ge \frac{a-b}{a-d}$ which completes the proof of (ii). In terms of (i) note that under $R = 0$ all $r^g > r^p$ satisfy $\frac{r^g}{r^p} \ge 2$. Using this jointly with $2 \ge \frac{a-b}{a-d}$ completes the proof. □

We explicitly formulate this result since it shows that credibly low inflation can obtain in equilibrium in a game theoretic setting that approaches the BG repeated game in the limit - even an infinitessimal amount of commitment may be sufficient.

6.2. **The Policymaker's Impatience.** While this section will show that the policymaker's impatience *worsens* coordination. Despite this we can still derive two main general results. The first one extends the convenient finding of Lemma 2 to the general BG game with the policymaker's impatience. While it only applies under sufficiently impatient public, this will be later argued to represent several realistic cases such as backward looking expectations or public's costly acquiring/processing information.

The second result extends the finding of Proposition 2 (showing that a sufficient degree of commitment always exists) from a fully patient policymaker to a sufficiently patient one (and for any discount factor of the public). These results are followed by several policy related findings that offer testable hypotheses.

**Theorem 2.** *Consider the general BG game in which* (1) *holds and* $0 \le \delta_p \le \overline{\delta_p} < 1$ *where* $\overline{\delta_p}$ *is some upper bound. Then* $\forall \delta_g, R$, *the sufficient condition to ensure only Ramsey SPNE relates to* $n^g = 1$ *(the initial simultaneous move).*

*Proof.* The proof of Theorem 1 showed that since disinflation is always costly under sufficiently impatient public, $\delta_p \le \overline{\delta_p}$, (15) no longer applies and (14) is the relevant condition $\forall k_{n+1}, R = (0, 1]$, and $\forall q, v, x, z$ satisfying (1). Under $\delta_g = 1$ (14) was shown in the proof of Lemma 2 to be the strongest at $n^g = 1$ for the general BG game. Under $\delta_g < 1$ (14) becomes

$$b \sum_{t=1}^{k_n} \delta_g^{t-1} + a \sum_{t=k_n+1}^{r^g} \delta_g^{t-1} + a \sum_{t=r^g+1}^{r^g+k_{n+1}} \delta_g^{t-1} > d \sum_{t=1}^{r^g} \delta_g^{t-1} + b \sum_{t=r^g+1}^{r^g+k_{n+1}} \delta_g^{t-1}. \tag{24}$$

[28] Note that this is observationally equivalent to backward looking expectations which will be discussed.

[29] Note that Theorem 1 does not claim that any SPNE is Ramsey - therefore the inequalities have changed from strict to weak.

Appendix A shows that the condition in (24) is again sufficient at $n^g = 1$. $\square$

This property means that regardless of the exact dynamics, it suffices to focus on the initial simultaneous move (similarly to a one-shot game) *assuming* that all further relevant conditions hold.[30] If the strongest condition for $n^g = 1$ is satisfied we then know that a unique (and efficient) equilibrium outcome obtains throughout.

**Theorem 3.** *Consider the general BG game in which* (1) *holds and the policymaker is sufficiently patient,*

$$\delta_g > \overline{\delta_g} = \sqrt[r^p]{1 - \frac{a-d}{c-b}}. \tag{25}$$

*Then there exists* $\overline{r^g} \in \mathbb{N}$ *such that for all* $r^g > \overline{r^g}$ *and* $\forall R, r^p, \delta_p$ *any SPNE of the game is Ramsey.*[31]

*Proof.* In terms of the proof first realize that (25) yields $0 < \overline{\delta_g} < 1$ for all assumed values. First, the inequality $|a-d| < |c-b|$ (direct consequence of (1)) ensures that the argument of the square root is positive. Moreover, the inequalities $a > d$ and $c > b$ imply that this argument is less than one.

We have shown in Theorem 1 that the public's impatience weakens the sufficient conditions. Therefore, it suffices to depict the worst case, $\delta_p = 1$. The proof of Proposition 3 showed that (15) combined with $k_n = r^p$ and $k_{n+1} = 0$ (resulting in (23)) is the general sufficient condition. Under $\delta_g < 1$, (15) becomes

$$b\sum_{t=1}^{k_n} \delta_g^{t-1} + a \sum_{t=k_n+1}^{r^g} \delta_g^{t-1} > d \sum_{t=1}^{r^g-r^p+k_{n+1}} \delta_g^{t-1} + c \sum_{t=r^g-r^p+k_{n+1}+1}^{r^g} \delta_g^{t-1}. \tag{26}$$

Now use the fact that this condition is the strongest for $k_n = r^p$ and $k_{n+1} = 0$ and rearrange (26) into

$$(a-d) \sum_{t=r^p+1}^{r^g-r^p} \delta_g^{t-1} > (d-b)\sum_{t=1}^{r^p} \delta_g^{t-1} + (c-a) \sum_{t=r^g-r^p+1}^{r^g} \delta_g^{t-1}. \tag{27}$$

Therefore, in this proof it suffices to show that (25) implies (27) which we do in Appendix B. $\square$

To demonstrate, if we consider the constraints of the specific BG game, (6), the expression in (25) becomes

$$\delta_g > \overline{\delta_g} = \sqrt[r^p]{\frac{2}{3}}.$$

We can analyze the dependence of $\overline{\delta_g}$ on the other variables (similarly to Proposition 3 in which we do so for $\overline{r^g}$). The fact that they all affect both of these thresholds in the same direction will be formalized later.

**Corollary 1.** *The sufficient threshold* $\overline{\delta_g}$ *in Theorem 3 is positively related to wage rigidity,* $r^p$*, the policymaker's temptation,* $c$*, and the disinflation cost,* $-b$*, and negatively related to the credibility benefit,* $a$*, and the inflation cost,* $-d$*.*

---

[30]Specifically, using $k_1$ and $k_2$ with (10), (14), and (15) implies that anything that happens in periods $t > r^g + r^p(1-R)$ can be 'skipped'.

[31]The Theorem implies that the necessary bound $\overline{r^g}$ depends on $\delta_g$. But for the sake of expositional clarity we use $\overline{r^g}$ instead of $\overline{r^g}(\delta_g)$ (and similarly for all thresholds in the rest of this section). This also implies that the conditions $r^g > \overline{r^g}$ and $\delta_g > \overline{\delta_g}$ are not sufficient for the existence of the Ramsey SPNE individually - they have to hold jointly.

While Theorem 3 reports the sufficient bound $\overline{\delta_g}$ it does not provide a sufficient commitment level $\overline{r^g}$ - it only shows its existence. This is because Theorem 3 is proven regardless of the value of $R$. But we have seen in Proposition 1 that (i) the value of $R$ determines the exact dynamics and that therefore (ii) the necessary and sufficient commitment level is a function of $R$ - ie $\widehat{r^g}(R)$ using the notation introduced in section 4.2.[32] Obviously, $\widehat{r^g}(R) \leq \overline{r^g}$.

Proposition 1 showed that under $\delta_g = \delta_p = 1$, thresholds $\widehat{r^g}(R)$ for $R = (0,1)$ differ quantitatively from $\widehat{r^g}(0)$ but are qualitatively the same. Therefore we investigate $\widehat{r^g}(0)$ and extend our conclusions to the remaining cases. Under the policymaker's impatience the condition analogous to (10) becomes

$$b\sum_{t=1}^{r^p}\delta_g^{t-1} + a\sum_{t=r^p+1}^{r^g}\delta_g^{t-1} > d\sum_{t=1}^{r^g}\delta_g^{t-1}. \tag{28}$$

which can be rewritten into

$$b\frac{1-\delta_g^{r^p}}{1-\delta_g} + a\delta^{r^p}\frac{1-\delta_g^{r^g-r^p}}{1-\delta_g} > d\frac{1-\delta_g^{r^g}}{1-\delta_g}.$$

By analyzing this equation we observe that (28) holds if and only if

$$\delta_g > \widehat{\delta}_g(0) = \sqrt[r^p]{\frac{d-b}{a-b}} \overset{(6)}{=} \sqrt[r^p]{0.5}, \tag{29}$$

and simultaneously

$$r^g > \widehat{r^g}(0) = \log_{\delta_g}\left(\frac{a-b}{a-d}\delta_g^{r^p} - \frac{d-b}{a-d}\right) \overset{(6)}{=} \log_{\delta_g}(2\delta_g^{r^p}-1). \tag{30}$$

We emphasize two properties which follow from (30). First, the argument of the logarithm in (30) is positive if and only if (29) holds. Second, both the base and the argument of the logarithm in (30) are then (strictly) between 0 and 1 - to see this realize that

$$0 < \frac{a-b}{a-d}\delta_g^{r^p} - \frac{d-b}{a-d} < \frac{a-b}{a-d} - \frac{d-b}{a-d} = 1.$$

Therefore $\widehat{r^g}(0)$ is positive and increasing in $r^p$ as in Proposition 3 (Figure 4 demonstrates the relationship between these variables graphically for various values of $\delta_g$).

It can therefore be argued that (i) the intuition of the patient policymaker environment is unchanged and that (ii) the existence result of Theorem 3 is fairly robust to the policymaker's discounting.[33] Furthermore, the following result about the relationship between $r^g$ and $\delta_g$ is apparent.

**Proposition 4.** *Consider the general BG game in which* (1) *holds and* $\delta_g > \widehat{\delta}_g(R)$. *Then the policymaker's patience,* $\delta_g$, *and commitment,* $r^g$, *are substitutes in achieving the Ramsey SPNE.*

*Proof.* In Appendix C we prove formally what Figure 4 shows graphically - that $\widehat{r^g}(R)$ from (30) is decreasing in $\delta_g$. □

[32] To document the difference between $\overline{r^g}$ and $\widehat{r^g}(R)$ recall the findings of Proposition 1. In the specific BG game with $\delta_g = \delta_p = 1$ we have $\widehat{r^g}(R) = \begin{cases} 2r^p & \text{if } R = 0, \\ 2Rr^p & \text{if } R \geq 0.5 \\ 2(1+R)r^p & \text{if } R < 0.5 \end{cases}$, see (11), (19), and (20) respectively. The last case is the strongest condition and implies that the sufficient level, independent of $R$, is $\overline{r^g} = \frac{5}{2}r^p$.

[33] For example, Figure 4 reports the following for the specific game (6). If $r^p = 1$ (which implies $R = 0$, $\forall r^g$) then under $\delta_g = 1$ we have $r^g > \widehat{r^g}(0) = 2$ (from (11)), under $\delta_g = 0.99$ we have $r^g > \widehat{r^g}(0) \approx 2.01$, under $\delta_g = 0.8$ we have $r^g > \widehat{r^g}(0) \approx 2.29$, and under $\delta_g = 0.51$ we have $r^g > \widehat{r^g}(0) \approx 5.81$ (all from (30)). Put differently, f $r^p = 1$ then for all $\delta_g > \overline{\delta_1} \approx 0.62$ the value $\overline{r^g} = 3$ suffices to ensure the Ramsey SPNE. Similarly, for $\delta_g > \overline{\delta_2} \approx 0.54$ the value $\overline{r^g} = 4$ suffices etc.

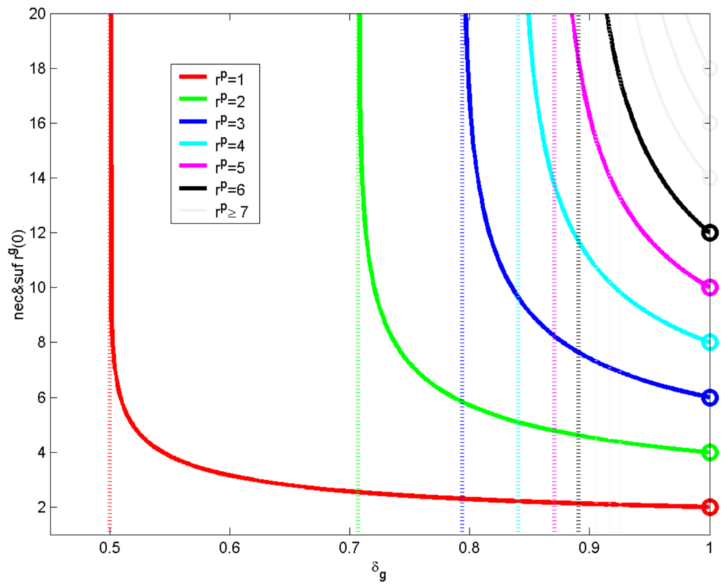



FIGURE 4. Dependence of $\widehat{r^g}(0)$ on $\delta_g$ for various $r^p$ (from (29) and (30) for the specific game (6)). Dotted asymptotes correspond to bounds $\widehat{\delta_g}(0)$ for each particular $r^p$.

This implies that a less patient policymaker needs to more strongly commit (make its IT more explicit) to ensure credibility of low inflation. The following corollaries summarize the adverse consequences of an opposite scenario - insufficient commitment.

**Corollary 2.** *Consider the general BG game in which* (1) *holds. If* $r^g$ *satisfies*

$$1 \leq r^g \leq \widehat{r^g}(R), \tag{31}$$

*then* $\pi^L$ *is time-inconsistent and, compared to the case* $r^g > \widehat{r^g}(R)$*, the average level of inflation is higher.*

Since $\widehat{r^g}(R)$ is the necessary and sufficient threshold and it is increasing in $r^p$ (see Proposition 3 or equation (30)) it follows that under $r^g \leq \widehat{r^g}(R)$ the level $\pi^H$ obtains on the equilibrium path for at least one $n^g$. This increases average inflation. The fact that $\widehat{r^g}(R) \geq r^p \geq 1, \forall R$ is implied by Theorems 1 and 3. Specifically, realize that (i) if $r^g = r^p$ (which is the standard repeated game) then $H$ will be played uniquely in equilibrium in all moves, $(i_n^H)^*, \forall i, n$. Further, (ii) if $r^g < r^p$ then $\forall R$, $H$ will be played in at least one move on the equilibrium path (the easiest way to see this is to note that the policymaker has the final move which will uniquely be $\pi^H$).

**Corollary 3.** *Consider the general BG game in which* (1) *holds. Further, assume that* $\delta_g > \widehat{\delta_g}(R)$ *and that* $r^g$ *and* $R = (0,1)$ *satisfy*

$$1 \leq \underline{r^g}(R) < r^g \leq \widehat{r^g}(R). \tag{32}$$

*where* $\underline{r^g}(R)$ *is some lower bound. Then, and only then, inflation variability is higher than under* $r^g > \widehat{r^g}(R)$*.*

Obviously, if $r^g > \widehat{r^g}(R)$ then the optimal $L$ level of (long-run) inflation obtains uniquely on the equilibrium path, ie the variability of (long-run) inflation is zero. In contrast, under the circumstances of Corollary 3 (if there exists $r^g$ satisfying (32)), both $\pi^L$ and $\pi^H$ obtain on the equilibrium path which increases inflation variability. Intuitively, due to $r^g \leq \widehat{r^g}(R)$ the policymaker inflates in at least one $n^g$ but due to $r^g > \underline{r^g}(R)$ and $\delta_g > \widehat{\delta_g}(R)$ he finds it optimal to eventually disinflate - for an example with $r^g > r^p$ see footnote 27 that describes the case $r^g = 4, r^p = 3$. Note however that (32) also includes cases of $r^g < r^p$. For example in the specific BG game with $\delta_g = \delta_p = 1$, under $r^g = 4, r^p = 5$ both players play $H$ throughout on the equilibrium path with the exception of their second moves in which they play uniquely $L$.

It is interesting to note that this result obtains even in the absence of shocks, ie for trend/long-run inflation. This arises because the gains and costs of inflating vary in time with $k_n$. We would also like to point out that there exist circumstances under which Corollary 2 applies but Corollary 3 does not. For example for all $r^g$ and $r^p$ such that $R = 0$ there exist(s) SPNE that has/have *either $H$ or $L$* uniquely on the equilibrium path (but not their combination) so volatility may not be higher (for this reason (32) does not include the case $r^g = 1$).

## 7. Empirical Evidence

Our analysis has several testable implications. The level of inflation and its variability are shown to be weakly decreasing (and the policy's credibility increasing) in the degree of the policymaker's: (i) long-run commitment, $r^g$, (ii) patience, $\delta_g$, (Corollaries 2 and 3 combined with Theorem 3) and (iii) conservatism, $\alpha$, (implied by (5) in the spirit of Rogoff (1985)). Further and interestingly, it implies a negative relationship between patience/conservatism on one hand and commitment on the other due to their substitutability (Proposition 4).

We will first discuss suitable proxies of these variables, then examine the patience-commitment relationship, and then revisit their effect on inflation and its variability. In doing the latter some conflicting empirical findings of the literature will be reconciled based on our theoretical results.

7.1. **Proxies.** In terms of (i) the policymaker's (long-run) commitment was interpreted above as the degree of explicitness of the IT. This is because the more explicitly is the long-run IT stated in the legislation the less frequently it can be altered.[34] While there exist no index that would measure the target's explicitness the closest proxies are arguable the pivotal features of the regime, namely the degrees of (goal) transparency and accountability that make the target rigid.

In terms of (ii) and (iii) it can be argued that patience and conservatism are a function of several characteristics of monetary policy, most importantly the degree of central bank goal-independence (goal-CBI).[35] First, goal-independent central bankers are commonly more conservative (tougher on inflation) in the spirit of Rogoff (1985). Second, they have a longer term in office which is likely to translate into more patient behaviour (see eg Eggertsson and Le Borgne (2003)). In the past two decades the real world has seen a move in the direction of increasing goal-CBI and greater length of the banker's term has come as one of the arrangements (see for example Waller and Walsh (1996)).[36]

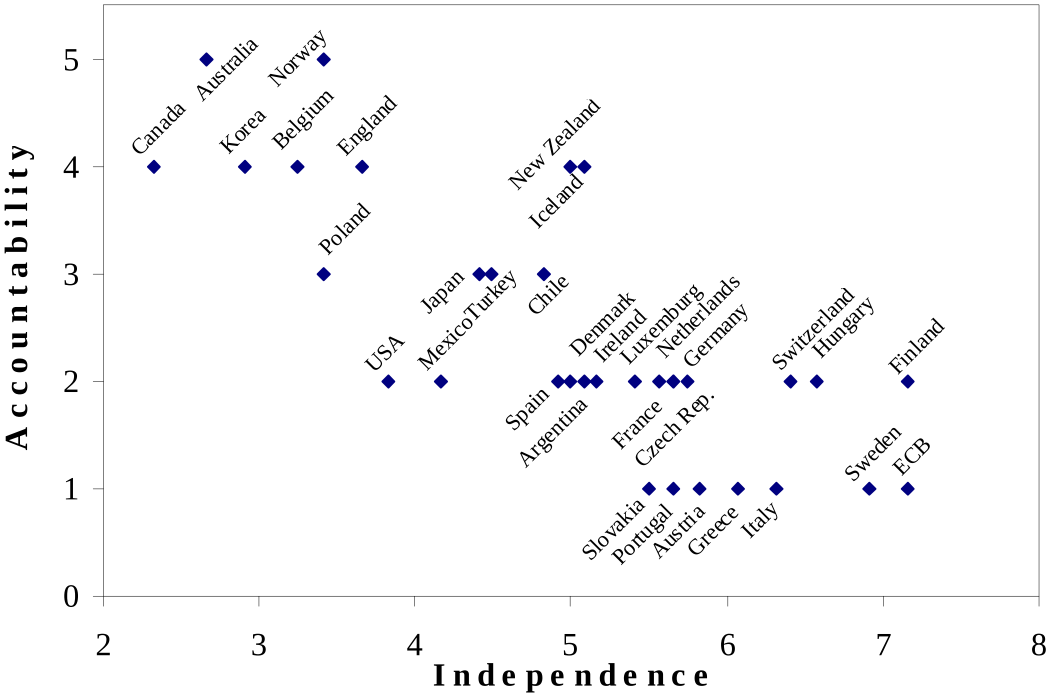
---

[34]To document, there have been only very few occasions over the past two decades whereby the level of an explicit IT has been altered (and the changes have been arguably trivial). Furthermore, no country to our knowledge has abandoned an explicit IT.

[35]It should be stressed that our paper makes prediction about the *goal*-CBI, not *instrument*-CBI (on this distinction see Debelle and Fischer (1994)). This is because both $\alpha$ and $\delta_g$ relate to the parameters in the policymaker's objective function.

[36]On the length in office for 93 countries see Mahadeva and Sterne (2000), Table 4.4. While the norm of 5-7 years is only marginally longer than the policymaker's term, in the majority of cases (in industrial countries) the

7.2. **Institutional Relationships.** Using these proxies implies that there exists substitutability between explicit inflation targeting and goal-CBI in ensuring low inflation and high credibility. This novel prediction is supported by several studies that report a negative correlation between (goal) CBI and accountability, eg Briault, Haldane and King (1997), de Haan, Amtenbrink and Eijffinger (1999), and Sousa (2002) (see Figure 5 for an example).




Central bank accountability versus independence, Sousa (2002). We use the 'final responsibility' component of accountability, see Appendix for details on the criteria, countries, and scores. The correlation coefficient equals −0.78 (the t value equals −6.94).

Despite the arguable shortcomings of any such index this finding seems robust as it has been obtaind using differently constructed indices for different countries and periods.[37] If we plot Sousa (2002) final responsibility against the length of term in office (which is one of the criteria in his CBI index) the picture remains roughly the same. Furthermore, in a comprehensive dataset of Fry et al. (2000) the length of term in office is negatively correlated to accountability procedures (that apply when targets are missed or must be changed) in both industrial and transition countries. Finally, Hughes Hallett and Libich (2006a) present evidence that transparency, too, is negatively correlated to goal-CBI. For example, it is shown that the correlation between transparency in

Governor gets reappointed which makes the expected term in office significantly longer. The U.S. offers itself as a good example.

[37] Note that while all top left hand corner countries are explicit inflation targeters, not all inflation targeters are in that corner – which is likely to be due to country specific factors.

Eijffinger and Geraats (2006) and goal-CBI in Briault, Haldane and King (1997) is −0.86 (and the $t$-value equals −4.46).[38]

7.3. **Effect on Inflation.** For the purposes of empirical testing it is important to note the exact nature of our results. The analysis implies that a more explicit long-run IT reduces the level of inflation and its the volatility, but only if the initial level of explicitness is insufficient to achieve the Ramsey SPNE, $r^g < \widehat{r^g}$ (see Corollaries 2 and 3). Otherwise $r^g$ may have no long-run effect.[39] Therefore, the results are not equivalent to the claim that IT countries will have lower inflation and its volatility than non-IT countries. This is because the latter group's implicit IT may still be sufficiently explicit, ie in the region of $r^g > \widehat{r^g}$.

Our analysis implies a criterion to distinguish whether this is or isn't the case - it suggests to examine the *average* level of inflation (say over the past 5 years), $\bar{\pi}$. If $\bar{\pi} > \pi^L$ (arguably the case of many transition and developing countries) then $r^g < \widehat{r^g}$ is implied and empirical analysis of such sample will find the explicitness of inflation targeting to be negatively correlated with both the level of inflation and its volatility. In contrast, if $\bar{\pi} = \pi^L$ (arguably the case of most industrial countries) then $r^g > \widehat{r^g}$ is implied and our model predicts no correlation. Both predictions are supported; papers that only include industrial countries find weak and/or insignificant effects of inflation targeting on inflation and its volatility, eg Ball and Sheridan (2003) and Willard (2006) whereas those with larger samples find strong and significant effects, eg Corbo, Landerretche and Schmidt-Hebbel (2001).

Furthermore, in line with the predictions of our model, inflation has been found negatively correlated with accountability (Briault, Haldane and King (1997)) as well as with transparency (Chortareas, Stasavage and Sterne (2002) and Fry et al. (2000)). See also Debelle (1997) who finds inflation targeting to increase the policy's credibility. All these papers include either pre-1980 inflation data or emerging/developing countries. In contrast, papers that only focus on industrial countries and use recent data often find no correlation, see eg Eijffinger and Geraats (2006).

Similarly, goal-CBI was found to be associated with lower inflation, see Grilli, Masciandaro and Tabellini (1991), Cukierman, Webb and Neyapti (1992), Alesina and Summers (1993), Eijffinger, Schaling and Hoeberichts (1998). However, using more recent data inflation is uncorrelated to goal-CBI among industrial countries, see eg Fry et al. (2000).

## 8. Robustness and Extensions

This section discusses some extensions and implies that our results are robust to a number of alternative specifications and assumptions.

8.1. **Macroeconomic Issues. Substitutability.** The negative relationship between goal-CBI and the explicitness of the IT is derived in Hughes Hallett and Libich (2006b) through an entirely different avenue. The paper uses a standard simultaneously repeated game but explicitly incorporates these features in the macroeconomic model.

**Short Run Stabilization.** As the paper takes a long-run view it is imperative to consider whether the findings are qualified in the presence of shocks. This is because some inflation

[38]For welfare implications of these institutional features and a more detailed empirical analysis see Hughes Hallett and Libich (2006a,b). The paper also demonstrates that the Debelle and Fischer (1994) distinction between goal and instrument CBI is crucial. Since instrument CBI has come hand in hand with inflation targeting (as one of the prerequisites of the regime, see eg Masson, Savastano and Sharma (1997) or Blejer and et al. (2002)) its correlation with transparency and accountability in most indices is positive, see eg Chortareas, Stasavage and Sterne (2002).

[39]Below it will be argued, based on Libich (2006b), that in the presence of shocks (ie in te short run) there may be an additional, anchoring, effect of an IT.

targeting opponents (see eg Kohn (2003), Friedman (2004) or Greenspan (2003)) have expressed concerns that a legislated numerical IT may reduce the policymaker's flexibility to react to shocks and stabilize output.

Our companion paper Libich (2006b) utilizes the rigid framework to investigate these concerns in detail. It uses a 'stochastic' New Keynesian type environment and a standard quadratic objective function. It shows that allowing for disturbances does not alter the conclusions of the presented paper if the IT is specified as a long-run objective (achievable on average over the business cycle).[40]

**Sticky Expectations.** While the analysis examined rigidity in wage setting the insights also apply to the public's (infrequent) updating of expectations. Libich (2006b) explicitly models this in the rigid framework by incorporating a cost of updating expectations (processing information). This goes in the spirit of the models of 'rational inattention' (see eg Sims (2003), Reis (2006)).

**Adaptive Expectations.** There is a large body of empirical research showing that backward looking expectations are important (see eg Fuhrer (1997)). Libich (2006a) considers a simple case of adaptive/static expectations in the rigid framework and shows that the outcomes of such static behaviour are equivalent to those under sufficiently impatient public studied above, $\delta_p < \overline{\delta_g}$ - in both cases the public will disregard the policymaker's future periods' play. This implies that the findings of Theorems 1 and 2 can be interpreted more generally as also applying to situations in which the public adopts a simple punishing rule of thumb of the 'tit-for-tat' variety (as in Barro and Gordon (1983)). The analysis further implies that adopting such a rule may be optimal for the public, even more so if processing information is costly which we discusse in the next section.

8.2. **Game Theoretic Issues. Generality**. In our companion work, Libich and Stehlik (2007), we examine other classes of games (some of which are used in macroeconomics as they describe the nature of certain real world strategic situations). It is shown that the intuition is unchanged; most importantly, the conditions can be derived under which in games with multiple Nash equilibria the efficient outcome is uniquely selected by subgame perfection. For example, the Battle of Sexes well describes the interaction between monetary and fiscal policy (see Hughes Hallett, Libich, and Stehlik (2006) and can be summarized by our general constraints in (1) with two modifications, $c < a$ and $z > q$. The above proofs imply that the results of Theorems 1-3 still obtain and some apply for a larger parameter space.

**Endogenous $r_m^i$.** It should be noted that all $r_m^i$'s can be endogenized as players' optimal choices. This seems desirable - while rigidity has been found important most common macroeconomic models take it as given.[41] Libich (2006a) is a step in this direction - it formalizes the concept of 'economically rational expectations' (Feige and Pearce (1976)) by incorporating various realistic costs into the players' objectives (that are some function of $r_m^i$) and letting them choose their $r_m^i$'s optimally (at the beginning of the game). In terms of the public it postulates a wage bargaining cost and a cost of updating expectations (processing information) about some stochastic process (shock). In terms of the policymaker a cost of explicit commitment is considered (such as implementation or accountability cost).

**Probabilistic $r_m^i$.** Deterministic commitment of Taylor (1980) can be reinterpreted as a probabilistic one in the spirit of the Calvo (1983) - setup 2 in section 2. In such case the average/expected length of time between each move is $\frac{1}{1-\theta^{i,m}}$ which is equivalent to our deterministic

[40]The paper in fact finds the opposite, the policymaker's flexibility under an explicit long-run IT is likely to increase which reduces the volatility of both inflation and output in equilibrium. This is due to the 'anchoring' effect of ITs that has been found empirically (eg Gurkaynak et al (2005)) and that our rigid framework enables us to model explicitly (more details on this are below). For arguments and results in the same spirit see Orphanides and Williams (2005), Bernanke (2003), Goodfriend (2003) and Mishkin (2004).

[41]Hahn is one of the notable exceptions in endogenizes price rigidity in the New Keynesian framework.

$r_m^i$. In a companion paper Libich and Stehlik (2007) we examine this probabilistic version explicitly. We show that the intuition remains the same, ie under a sufficiently committed and patient policymaker, $\theta^g > \overline{\theta^g}$ and $\delta_g > \overline{\delta_g}$, the Ramsey SPNE obtains.

**More Players - eg Heterogeneous Public.** The number of players can easily be increased - for example, Libich (2006b) models heterogeneous public (that may not bargain wages collectively). The players' set is then $I = \{g, p^j\}$ where $j \in [1, J]$ denotes a certain Union (individual) with wage rigidity of $r_j^p$ and relative size $P_j$ such that $\sum_1^J P_j = 1$. The paper shows that the necessary condition of the specific BG game for the case $R = 0$ equivalent to Proposition 1, (11), generalizes from $r^g > 2r^p$ to

$$r^g > 2\sum_{j=1}^{J} P_j r_j^p. \tag{33}$$

This demonstrates that the nature of the results remains unchanged.[42]

**More Instruments.** Each player can have a number of choice variables, each with a certain degree of rigidity/commitment. For example, Libich (2006b) models the policymaker as having two instruments - it sets the level of the long-run IT every $r_T^g$ periods and in addition it selects the short term interest rate every $r_i^g$ periods.[43]

**Continuous Time.** Libich and Stehlik (2007) present analogous results for continuous time, $t \in \mathbb{R}$, which can incorporate not only the players heterogenity but also the probablistic models. Roughly speaking, if we denote by $f : [0, r^g] \to [0, 1]$ an non-decreasing function which describes a (possibly probabilistic) distribution of the public's (Unions') reactions, then the condition analogous to (11), $r^g > 2r^p$, is

$$\int_0^{r^g} f(t)dt > \frac{r^g}{2}. \tag{34}$$

**Time Scales.** Both continuous and discrete models can be illustratively generalised using time scales (a recent mathematical tool see eg Bohner and Peterson (2001) for a comprehensive treatment). It enables us to neatly unify and extend all of the above mentioned setups and results. A *time scale* $\mathbb{T}$ is defined as a nonempty closed subset of the real numbers $\mathbb{R}$. In the analysis, the so-called 'jump operators' play a key role. The main contribution of this environment is the ability to consider *non-constant* (heterogeneous) rigidity/commitment. This generalization is arguably realistic and hence important in many settings in economics, econometrics, as well as other disciplines.[44] Libich and Stehlik (2007) show that the condition analogous to (11) and (34) is

$$\int_0^{r^g} f(t)\Delta t > \frac{r^g}{2}. \tag{35}$$

where the LHS is called 'delta integral' such that

[42] For example, with three equally sized Unions the condition becomes $\frac{5}{6}(r_1^p + r_2^p + r_3^p)$.

[43] The paper shows that while these instruments are not directly dependent there is an indirect link through the behaviour of the public. In particular, it is shown that the public's optimally chosen frequency of revising wages and expectations is a decreasing function of the target's explicitness, $(r^p)^* = f(\overset{+}{r_T^g})$. This 'anchoring effect' then leads to the interest rate instrument being more effective not only in stabilization of inflation but also of output - the opposite of what IT opponents argue.

[44] For an interesting application of time scales in economics see Biles, Atici and Lebedinsky (2005). The authors model payments to an agent (eg capital income or dividents) arriving an unevenly spaced intervals.

$$\int_0^{r^g} f(t)\Delta t = \begin{cases} \int_0^{r^g} f(t)dt & \text{if} \quad \mathbb{T} \in \mathbb{R}, \\ \sum_{t=0}^{r^g-1} f(t) & \text{if} \quad \mathbb{T} \in \mathbb{Z}. \end{cases} \tag{36}$$

This shows that time scale calculus, while nesting both continuous and discrete time as special cases, allows for even more flexible analysis of *dynamic interactions* with heterogeneous time steps.

## 9. Summary and Conclusions

This paper proposes a simple framework that generalizes the timing structure of macroeconomic (as well as other) games. As most such real world games are arguably finite, dynamic and most importantly, rigid, our framework combines these characteristics. We show that, similarly to reputation in repeated games, players' rigidity draws a link between successive periods and can therefore serve as a commitment device. This can enhance cooperation and eliminate inefficient outcomes from the set possible equilibria.

We apply the framework to the influential Barro-Gordon game and show the conditions under which the inflation bias disappears. Specifically we derive the exact degree of policy commitment that makes low inflation time-consistent and credible.

It is interesting to note that (i) this can happen in a finite game (possibly as short as two periods), (ii) the required levels of commitment may be rather (even infinitesimally) low and (iii) the policy commitment may substitute goal-CBI (conservatism and/or patience) in achieving credibility.

These results offer an explanation for the convergence to low inflation and high credibility in industrial countries over the past two decades - as a consequence of explicit inflation targeting and CBI. Furthermore, their substitutability helps explain why inflation targets have been made more explicit by countries with low degree of goal-CBI such as New Zealand, Canada, and the UK, than the relatively goal-independent central banks in the US, Germany, and Switzerland. We not only bring empirical support for all our findings but also reconcile some conficting results of the existing literature.

## 10. References

BACKUS, D., and J. DRIFFILL (1985): "Rational Expectations and Policy Credibility Following a Change in Regime," Review of Economic Studies, 52, 211-21.

BALL, L. (2000): "Near-Rationality and Inflation in Two Monetary Regimes," Johns Hopkins - Department of Economics Johns Hopkins - Department of Economics.

BALL, L., and N. SHERIDAN (2003): "Does Inflation Targeting Matter?," International Monetary Fund IMF Working Papers: 03/129.

BARRO, R. J., and D. B. GORDON (1983): "Rules, Discretion and Reputation in a Model of Monetary Policy," Journal of Monetary Economics, 12, 101-21.

BERNANKE, B. S., T. LAUBACH, F. S. MISHKIN, and A. POSEN (1999): Inflation Targeting: Lessons from the International Experience. Princeton: Princeton University Press.

BERNANKE, B. S., and M. WOODFORD (2005): "The Inflation-Targeting Debate," NBER Studies in Business Cycles, vol. 32. Chicago and London: University of Chicago Press, ix, 458.

BEWLEY, T. F. (2002): "Fairness, Reciprocity, and Wage Rigidity," Cowles Foundation Yale University Cowles Foundation Discussion Papers: 1383, 36 pages.

BHASKAR, V. (2002): "On Endogenously Staggered Prices," Review of Economic Studies, 69, 97-116.

BILES, D. C., F. M. ATICI, and A. LEBEDINSKY (2005): "An Application of Time Scales to Economics," Mathematical and Computer Modelling.

BLEJER, M. I., A. IZE, A. M. LEONE, and S. WERLANG (2000): Inflation Targeting in Practice: Strategic and Operational Issues and Application to Emerging Market Countries. International Monetary Fund.

BOHNER, M., and A. PETERSON (2001): Dynamic Equations on Time Scales. Birkhauser Boston, 704 Inc., Boston, MA, 2001.

BRIAULT, C., A. HALDANE, and M. KING (1997): "Independence and Accountability," in Towards More Effective Monetary Policy, ed. by I. e. Kuroda. London: Macmillan Press in association with Bank of Japan, 299-326.

CALVO, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics, 12, 383-98.

CANZONERI, M. B. (1985): "Monetary Policy Games and the Role of Private Information," American Economic Review, 75, 1056-70.

CARROLL, C. (2003): "Macroeconomic Expectations of Households and Professional Forecasters," Quarterly Journal of Economics, 118, 269-98.

CARROLL, C., and J. SLACALEK (2006): "Sticky Expectations and Consumption Dynamics."

CHO, I., and A. MATSUI (2005): "Time Consistency in Alternating Move Policy Games," Japanese Economic Review.

CHORTAREAS, G., D. STASAVAGE, and G. STERNE (2002): "Does It Pay to Be Transparent? International Evidence from Central Bank Forecasts," Federal Reserve Bank of St. Louis Review, 84, 99-117.

CORBO, V., O. LANDERRETCHE, and K. SCHMIDT-HEBBEL (2001): "Assessing Inflation Targeting after a Decade of World Experience," International Journal of Finance and Economics, 6, 343-68.

CUKIERMAN, A., S. B. WEBB, and B. NEYAPTI (1992): "Measuring the Independence of Central Banks and Its Effect on Policy Outcomes," World Bank Economic Review, 6, 353-98.

DE HAAN, J., F. AMTENBRINK, and S. C. W. EIJFFINGER (1999): "Accountability of Central Banks: Aspects and Quantification," Banca Nazionale del Lavoro Quarterly Review, 52, 169-93.

DEBELLE, G. (1997): "Inflation Targeting in Practice," International Monetary Fund IMF Working Papers: 97/35.

DEBELLE, G., and S. FISCHER (1994): "How Independent Should a Central Bank Be?," in Goals, Guidelines, and Constraints Facing Monetary Policymakers: FRB of Boston, Conference Series No. 38.

EGGERTSSON, G. B., and E. LE BORGNE (2003): "A Political Agency Theory of Central Bank Independence," International Monetary Fund IMF Working Papers: 03/144.

EIJFFINGER, S., E. SCHALING, and M. HOEBERICHTS (1998): "Central Bank Independence: A Sensitivity Analysis," European Journal of Political Economy, 14, 73-88.

EIJFFINGER, S. C. W., and P. M. GERAATS (2006): "How Transparent Are Central Banks?," European Journal of Political Economy, (forthcoming).

FEIGE, E. L., and D. K. PEARCE (1976): "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?," The Journal of Political Economy, Vol. 84, No. 3, 499-522. .

FISCHER, S. (1977): "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy, 85, 191-205.

FRIEDMAN, B. M. (2004): "Why the Federal Reserve Should Not Adopt Inflation Targeting," International Finance, 7, 129-36.

FRY, M., D. JULIUS, L. MAHADEVA, S. ROGER, and G. STERNE (2000): "Key Issues in the Choice of a Monetary Policy Framework," in Monetary Frameworks in a Global Context, ed. by L. Mahadeva, and G. Sterne. London: Routledge.

FUHRER, J. (1997): "The (Un)Importance of Forward Looking Behaviour in Price Specifications," Journal of Money, Credit and Banking, 29(3), 338-50.

GOODFRIEND, M. (2003): "Inflation Targeting in the United States?," National Bureau of Economic Research Inc NBER Working Papers: 9981.

GREENSPAN, A. (2003): "Monetary Policy and Uncertainty: Adapting to a Changing Economy, Symposium Sponsored by the Federal Reserve Bank of Kansas City," Jackson Hole, Wyoming, August 28 - 30, 2003.

GURKAYNAK, R. S., B. SACK, and E. SWANSON (2005): "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," American Economic Review, 95, 425-36.

HAAN, J. D., F. AMTENBRINK, and S. C. W. EIJFFINGER (1998): "Accountability of Central Banks : Aspects and Quantification," Tilburg University Center for Economic Research Discussion Paper: 54.

HAHN, V. (2006): "Endogenous Attentiveness of Price-Setters and Central Bank Conservatism," Heidelberg University, mimeo.

HUGHES HALLETT, A., and J. LIBICH (2006a): "Central Bank Independence, Accountability, and Transparency: Democratic Complements or Strategic Substitutes?," Centre for Economic Policy Research, London, CEPR DP 5470.

HUGHES HALLETT, A., and J. LIBICH (2006b): "Explicit Inflation Targets, Communication, and Central Bank Independence: Friends or Foes? ," mimeo, George Mason University.

HUGHES HALLETT, A., J. LIBICH, and P. STEHLIK (2006): "Various Types and Degrees of Monetary and Fiscal Commitment and Their Interaction," mimeo.

KOHN, D. L. (2003): "Remarks at the 28th Annual Policy Conference: Inflation Targeting: Prospects and Problems," Federal Reserve Bank of St. Louis, St. Louis, Missouri.

KYDLAND, F. E., and E. C. PRESCOTT (1977): "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy, 85, 473-91.

LAGUNOFF, R., and A. MATSUI (1997): "Asynchronous Choice in Repeated Coordination Games," Econometrica, 65, 1467-77.

LIBICH, J. (2006a): "An Explicit Inflation Target as a Commitment Device," Centre for Applied Macroeconomic Analysis, CAMA WP 22/2006.

LIBICH, J. (2006b): "Inflexibility of Inflation Targeting Revisited: Modelling the ‘Anchoring’ Effect," Centre for Applied Macroeconomic Analysis, CAMA WP 2/2006.

LIBICH, J., and P. STEHLIK (2007): "Rational Inattention in Games," mimeo, La Trobe University.

MANKIW, N. G., and R. REIS (2002): "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," Quarterly Journal of Economics, 117, 1295-328.

MASKIN, E., and J. TIROLE (1988): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," Econometrica, 56, 549-69.

MASKIN, E., and J. TIROLE (2001): "Markov Perfect Equilibrium: I. Observable Actions," Journal of Economic Theory, 100, 191-219.

MASSON, P. R., M. A. SAVASTANO, and S. SHARMA (1997): "The Scope for Inflation Targeting in Developing Countries," International Monetary Fund IMF Working Papers: 97/130.

MCCALLUM, B. T. (2003): "Inflation Targeting for the United States," Shadow Open Market Committee, available at http://www.somc.rochester.edu/May03/McCallum.pdf.

MISHKIN, F. S. (2004): "Why the Federal Reserve Should Adopt Inflation Targeting," International Finance, 7, 117-27.

MORRIS, S., and H. S. SHIN (2006): "Inertia of Forward-Looking Expectations," mimeo, Princeton University.

ORPHANIDES, A., and J. C. WILLIAMS (2005): "Imperfect Knowledge, Inflation Expectations, and Monetary Policy," Journal of Economic Dynamics and Control, Elsevier 29(11), 1807-8.

REIS, R. (2006): "Inattentive Consumers," Journal of Monetary Economics, 53 (8).

ROGOFF, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," Quarterly Journal of Economics, 100, 1169-89.

SIMS, C. A. (2003): "Implications of Rational Inattention," Journal of Monetary Economics, 50 (3), 665-90.

SOUSA, P. (2002): "Central Bank Independence and Democratic Accountability," mimeo, Portucalense University.

SVENSSON, L. E. O. (1999): "Inflation Targeting as a Monetary Policy Rule," Journal of Monetary Economics, 43, 607-54.

TAYLOR, J. B. (1979): "Staggered Wage Setting in a Macro Model," American Economic Review, 69, 108-13.

TAYLOR, J. B. (1980): "Aggregate Dynamics and Staggered Contracts," Journal of Political Economy, 88, 1-23.

TOBIN, J. (1982): "Money and Finance in the Macroeconomic Process," Journal of Money, Credit and Banking, 14 (2), 171-204.

WALLER, C. J., and C. E. WALSH (1996): "Central-Bank Independence, Economic Behavior, and Optimal Term Lengths," American Economic Review, 86, 1139-53.

WEN, Q. (2002): "Repeated Games with Asynchronous Moves," Department of Economics Vanderbilt University Working Papers: 0204.

WILLARD, L. (2006): "Does Inflation Targeting Matter: A Reassessment," CEPS WP 120, Princeton University.

WU, T. (2004): "Does Inflation Targeting Reduce Inflation? An Analysis for the Oecd Industrial Countries," Banco Central do Brazil Working Paper 83.

## Appendix A. Proof of Theorem 2

*Proof.* Equation (24) can be rearranged into

$$(d-b)\sum_{t=1}^{k_n}\delta_g^{t-1}-(a-d)\sum_{t=k_n+1}^{r^g}\delta_g^{t-1}-(a-b)\sum_{t=r^g+1}^{r^g+k_{n+1}}\delta_g^{t-1}<0.$$

Use $a-b=(a-d)+(d-b)$ and split the first series to obtain

$$(a-b)\sum_{t=1}^{k_n}\delta_g^{t-1}-(a-d)\sum_{t=1}^{r^g}\delta_g^{t-1}-(a-b)\sum_{t=r^g+1}^{r^g+k_{n+1}}\delta_g^{t-1}<0,$$

add $\sum_{t=k_n+1}^{r^g}\delta_g^{t-1}$ to both sides and collect the terms

$$(a-b)\sum_{t=1}^{r^g}\delta_g^{t-1}-(a-d)\sum_{t=1}^{r^g}\delta_g^{t-1}<(a-b)\sum_{t=k_n+1}^{r^g+k_{n+1}}\delta_g^{t-1}.$$

Adding up the series on the RHS we receive

$$(a-b)\sum_{t=1}^{r^g}\delta_g^{t-1}-(a-d)\sum_{t=1}^{r^g}\delta_g^{t-1}<(a-b)\delta_g^{k_n}\frac{1-\delta_g^{r^g+k_{n+1}-k_n}}{1-\delta_g}.$$

Since $\delta_g<1$ we see that the RHS is minimal (and therefore the inequality the most difficult to satisfy), when $k_n$ and $k_n-k_{n+1}$ are maximal. Since both $k_1=r^p$ and $k_1-k_2=Rr^p$ are maximal attainable values in our setup, we conclude that the sufficient condition for $R=(0,1]$ is related to $n^g=1$. Finally, we realize that for $R=0$ we have $N^g=1$ which finishes the proof. $\square$

## Appendix B. Proof of Theorem 3

*Proof.* First, let us realize that the assumptions of Theorem 3 imply

$$\delta_g^{r^p}>\frac{c-b+d-a}{c-b},$$

which is equivalent to

$$\frac{\delta_g^{r^p}}{1-\delta_g^{r^p}}>\frac{d-b+c-a}{a-d},$$

and can be rearranged into

$$0<1-\frac{d-b+c-a}{a-d}\frac{1-\delta_g^{r^p}}{\delta_g^{r^p}}.$$

Since $\delta_g^{2r^p}>0$, we have

$$0<\delta_g^{2r^p}\left(1-\frac{d-b+c-a}{a-d}\frac{1-\delta_g^{r^p}}{\delta_g^{r^p}}\right).$$

Consequently, for each $\delta_g = (0,1)$ there exists $\overline{r^g} \in \mathbb{N}$ such that for all $r^g > \overline{r^g}$

$$\delta_g^{r^g} < \delta_g^{2r^p}\left(1 - \frac{d-b+c-a}{a-d}\frac{1-\delta_g^{r^p}}{\delta_g^{r^p}}\right).$$

Multiplying both sides by $-(a-d)\delta_g^{r^p} > 0$ and dividing by $\delta_g^{2r^p}$ we obtain

$$(a-d)\delta_g^{r^p}\left(1-\delta_g^{r^g-2r^p}\right) > (d-b+c-a)(1-\delta_g^{r^p}).$$

Moreover, we divide both sides by $1-\delta_g > 0$ and split the right-hand side

$$(a-d)\delta_g^{r^p}\frac{1-\delta_g^{r^g-2r^p}}{1-\delta_g} > (d-b)\frac{1-\delta_g^{r^p}}{1-\delta_g} + (c-a)\frac{1-\delta_g^{r^p}}{1-\delta_g}.$$

Note that the three fractions are in fact partial sums of geometric series with quotient $\delta_g$

$$0 > (d-b)\sum_{t=1}^{r^p}\delta_g^{t-1} - (a-d)\sum_{t=r^p+1}^{r^g-r^p}\delta_g^{t-1} + (c-a)\sum_{t=1}^{r^p}\delta_g^{t-1}. \tag{37}$$

Finally, we realize that

$$\sum_{t=1}^{r^p}\delta_g^{t-1} > \sum_{t=r^g-r^p+1}^{r^g}\delta_g^{t-1},$$

which, in connection with $c-a>0$ and equation (37), implies that (27) holds. □

## Appendix C. Proof of Proposition 4

*Proof.* Take (30) and rewrite it into

$$\overline{r^g} = \frac{\ln(\frac{a-b}{a-d}\delta_g^{r^p} - \frac{d-b}{a-d})}{\ln\delta_g}.$$

Our task is to show that $\overline{r^g}$ is decreasing in $\delta_g$ on the considered domain

$$D := \left(\sqrt[r^p]{\frac{d-b}{a-b}}, 1\right). \tag{38}$$

For the sake of clarity, we simplify the notation by defining

$$\gamma := \frac{a-b}{a-d},\ \omega := \frac{d-b}{a-d},\ r := r^p, \delta := \delta_g.$$

Therefore, we want to show that the function

$$f(\delta) = \frac{\ln(\gamma\delta^r - \omega)}{\ln\delta}$$

is decreasing in $\delta$, or equivalently that $f'(\delta) < 0$ on $D$. Obviously,

$$\begin{aligned} f'(\delta) &= \frac{\frac{r\gamma\delta^{r-1}}{\gamma\delta^r-\omega}\ln\delta - \frac{1}{\delta}\ln(\gamma\delta^r-\omega)}{\ln^2\delta} \\ &= \frac{r\gamma\delta^r\ln\delta - (\gamma\delta^r-\omega)\ln(\gamma\delta^r-\omega)}{\delta(\gamma\delta^r-\omega)\ln^2\delta}. \end{aligned}$$

Since the denominator is always positive, it suffices to show that

$$r\gamma\delta^r\ln\delta - (\gamma\delta^r-\omega)\ln(\gamma\delta^r-\omega) < 0,$$

or equivalently

$$\phi(\delta) := r\gamma\delta^r \ln\delta < (\gamma\delta^r - \omega)\ln(\gamma\delta^r - \omega) =: \psi(\delta)$$

on the considered domain $D$ from (38). Taking into account definitions of $\gamma$ and $\omega$ we observe that $\phi(1) = 0 = \psi(1)$. Therefore it suffices to show that $\phi'(\delta) > \psi'(\delta)$ for all $\delta \in D$. But this is satisfied since:

$$\begin{aligned}
\phi'(\delta) &> \psi'(\delta)\\
r^2\gamma x^{r-1}\ln\delta + r\gamma x^{r-1} &> r\gamma x^{r-1}\ln(\gamma\delta^r - \omega) + r\gamma x^{r-1}\\
r\ln\delta &> \ln(\gamma\delta^r - \omega)\\
\delta^r &> \gamma\delta^r - \omega\\
\delta_g^{r^p} &> \frac{a-b}{a-d}\delta_g^{r^p} - \frac{d-b}{a-d}\\
(a-d)\delta_g^{r^p} &> (a-b)\delta_g^{r^p} - (d-b)\\
0 &> (d-b)(\delta_g^{r^p} - 1),
\end{aligned}$$

where the last inequality is trivially satisfied since $d > b$ and $\delta_g < 1$. □

## Appendix D. Central Bank Independence Index (Sousa, 2002)

Each of the following criteria is assigned up to one point, for more details see Sousa (2002).

| PERSONAL INDEPENDENCE |
|---|
| 1. Appointment of the central bank board members |
| 2. Mandate duration of more than half of the CB board members. |
| 3. Fiscal policymaker's participation at central bank meetings. |
| POLITICAL INDEPENDENCE |
| 4. Ultimate responsibility and authority on monetary policy decisions. |
| 5. Price stability |
| 6. Banking supervision |
| 7. Monetary policy instruments |
| ECONOMIC AND FINANCIAL INDEPENDENCE |
| 8. Policymaker financing |
| 9. Ownership of the central bank's (equity) capital |

## Appendix E. Central Bank Accountability Index (Sousa, 2002)

Criteria and methodology adopted from De Haan et al. (1998). We only use the 'final responsibility' component that best proxies the policymaker's LR commitment (explicitness of the IT). Each of the following criteria is assigned up to one point, for more details see Sousa (2002).

| FINAL RESPONSIBILITY |
|---|
| 1. Is the central bank subject of monitoring by Parliament? |
| 2. Has the policymaker (or Parliament) the right to give instruction? |
| 3. Is there some kind of review in the override procedure? |
| 4. Has CB possibility for an appeal in case of an instruction? |
| 5. Can the CB law be changed by a simple majority in Parliament? |
| 6. Is past performance a ground for dismissal of a central bank governor? |

## Appendix F. Evaluation Table

| Index | | Independence Sousa (2002)* | | | | Accountability Sousa (2002)* |
|---|---|---|---|---|---|---|
| | Country | Personal | Political | Economic / Financial | Total# | Final Responsibility |
| 1 | Argentina | 1.25 | 2.83 | 1.00 | 5.08 | 2 |
| 2 | Australia | 0.50 | 2.16 | 0.00 | 2.66 | 5 |
| 3 | Austria | 1.66 | 3.16 | 1.00 | 5.82 | 1 |
| 4 | Belgium | 1.75 | 1.50 | 0.00 | 3.25 | 4 |
| 5 | Canada | 0.50 | 1.83 | 0.00 | 2.33 | 4 |
| 6 | Chile | 2.00 | 1.83 | 1.00 | 4.83 | 3 |
| 7 | Czech Republic | 1.58 | 3.16 | 1.00 | 5.74 | 2 |
| 8 | Denmark | 2.16 | 2.83 | 0.00 | 4.99 | 2 |
| 9 | EMU - ECB | 2.50 | 3.66 | 1.00 | 7.16 | 1 |
| 10 | England | 1.00 | 2.66 | 0.00 | 3.66 | 4 |
| 11 | Finland | 2.50 | 3.66 | 1.00 | 7.16 | 2 |
| 12 | France | 1.50 | 3.16 | 1.00 | 5.66 | 2 |
| 13 | Germany | 1.50 | 3.16 | 1.00 | 5.66 | 2 |
| 14 | Greece | 1.91 | 3.16 | 1.00 | 6.07 | 1 |
| 15 | Hungary | 1.91 | 3.66 | 1.00 | 6.57 | 2 |
| 16 | Iceland | 1.75 | 3.33 | 0.00 | 5.08 | 4 |
| 17 | Ireland | 1.00 | 3.16 | 1.00 | 5.16 | 2 |
| 18 | Italy | 2.16 | 3.16 | 1.00 | 6.32 | 1 |
| 19 | Japan | 0.75 | 3.66 | 0.00 | 4.41 | 3 |
| 20 | Korea | 0.75 | 2.16 | 0.00 | 2.91 | 4 |
| 21 | Luxemburg | 1.25 | 3.16 | 1.00 | 5.41 | 2 |
| 22 | Mexico | 1.83 | 2.33 | 0.00 | 4.16 | 2 |
| 23 | Netherlands | 2.41 | 3.16 | 0.00 | 5.57 | 2 |
| 24 | New Zealand | 1.83 | 2.16 | 1.00 | 4.99 | 4 |
| 25 | Norway | 1.58 | 1.83 | 0.00 | 3.41 | 5 |
| 26 | Poland | 1.25 | 2.16 | 0.00 | 3.41 | 3 |
| 27 | Portugal | 1.50 | 3.16 | 1.00 | 5.66 | 1 |
| 28 | Slovakia | 1.00 | 3.50 | 1.00 | 5.50 | 1 |
| 29 | Spain | 0.75 | 3.16 | 1.00 | 4.91 | 2 |
| 30 | Sweden | 2.75 | 3.16 | 1.00 | 6.91 | 1 |
| 31 | Switzerland | 2.08 | 3.33 | 1.00 | 6.41 | 2 |
| 32 | Turkey | 1.66 | 2.83 | 0.00 | 4.49 | 3 |
| 33 | USA | 2.00 | 1.83 | 0.00 | 3.83 | 2 |

*Assessment is based on the situation in January 2002. # Excludes aspect 9 due to missing observations.